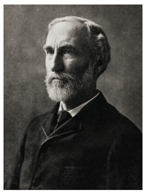
NAME: SOLUTION
AME 20231, Thermodynamics
Examination 1
Prof. J. M. Powers
11 February 2020



"Suppose the body to change its state, the points associated with the states through which the body passes will form a line, which we may call the *path* of the body."
Transactions of the Connecticut Academy, II., pp. 309-342, April-May 1873.

Josiah Williard Gibbs, b. 11 February 1839, d. 28 April 1903
Happy 181st Birthday, Prof. Gibbs!

1. (10) A box with $V = 1$ m^3 contains an ideal gas at $P = 1$ Pa and $T = 100$ K. Determine the number of moles in the box.

Solution

We use the ideal gas law

$$PV = n\overline{R}T.$$

Solve for n, the number of moles, to get

$$n = \frac{PV}{\overline{R}T} = \frac{(1 \text{ Pa})(1 \text{ m}^3)}{\left(8.3145 \frac{\text{J}}{\text{mol K}}\right)(100 \text{ K})} = \boxed{0.00120272 \text{ mol.}}$$

2. (10) A two-phase liquid-vapor mixture of H_2O at $v = 0.013$ m^3/kg and $T = 100°$C is heated isochorically until it is a single phase. Determine the final temperature and pressure of the H_2O. Determine if the final state is solid, liquid, or gas. Sketch the process in the $T - v$ plane.

Solution

From the tables, we see the critical volume $v_c = 0.00315$ m^3/kg. So we have $v > v_c$. If we heat isochorically under the dome, the temperature will rise until it reaches the saturated vapor dome. We see in the tables that that happens at

$$T = \boxed{330°\text{C}}, \qquad P = \boxed{12845 \text{ kPa.}}$$

The final state is $\boxed{\text{gas.}}$ The appropriate sketch is given here in

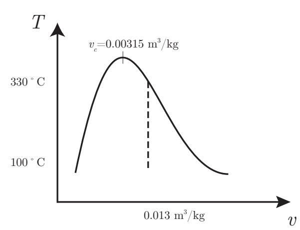


Figure 1: Sketch of isochoric heating of H_2O (not to scale).

3. (20) N_2 is at $P = 2121$ kPa, $T = 140$ K.

   (a) Find v with the ideal gas law.
   (b) Find v with the superheated nitrogen tables.
   (c) Find v with the compressibility chart, Fig. D.1.
   (d) Give an accurate sketch of the actual state of the N_2 in the $P-v$, $T-v$, and $P-T$ planes. The state should be properly placed relative to the vapor domes and critical points, which should also be part of the sketch.

Solution

First use the ideal gas law.

$$v = \frac{RT}{P} = \frac{\left(0.2968\ \frac{\text{kJ}}{\text{kg K}}\right)(140\text{ K})}{2121\text{ kPa}} = \boxed{0.0195908\ \frac{\text{m}^3}{\text{kg}}}.$$

We can interpolate Table B.6.2 between $P = 2000$ kPa and $P = 3000$ kPa. We get

$$v = \left(0.01752\ \frac{\text{m}^3}{\text{kg}}\right) + \left(\frac{0.01038\ \frac{\text{m}^3}{\text{kg}} - 0.01752\ \frac{\text{m}^3}{\text{kg}}}{(3000\text{ kPa}) - (2000\text{ kPa})}\right)((2121\text{ kPa}) - (2000\text{ kPa})) = \boxed{0.0166561\ \frac{\text{m}^3}{\text{kg}}}.$$

Next use the compressibility charts. We have $P_c = 3397.8$ kPa, $T_c = 126.2$ K. So

$$P_r = \frac{2121\text{ kPa}}{3397.8\text{ kPa}} = 0.624227, \qquad T_r = \frac{140\text{ K}}{126.2\text{ K}} = 1.10935.$$

Here we find $Z \sim 0.8$. Now $Z = Pv/R/T$, so

$$v = Z\frac{RT}{P} = 0.8\frac{\left(0.2968\ \frac{\text{kJ}}{\text{kg K}}\right)(140\text{ K})}{2121\text{ kPa}} = \boxed{0.0156726\ \frac{\text{m}^3}{\text{kg}}}.$$

The appropriate sketches are given here in

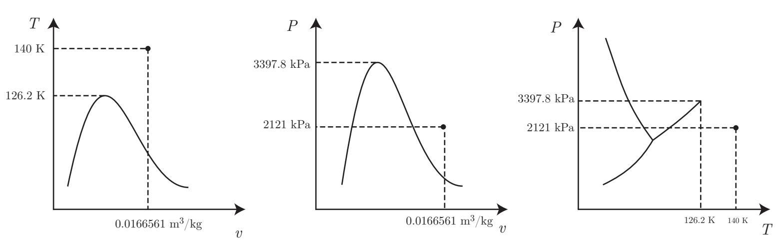


Figure 2: State of N_2 in $T - v$, $P - v$, and $P - T$ planes (not to scale).

4. (30) An ideal gas of mass m and gas constant R at pressure P_1 and temperature T_1 is compressed in an isochoric process until $T_2 = 2T_1$. The gas is isothermally expanded back to $P_3 = P_1$. The gas then undergoes an isobaric process to return to P_1 and T_1.

   (a) Find P_2 in terms of P_1, T_1, m, and R.
   (b) Find the work of each process in the cycle, ${}_1W_2$, ${}_2W_3$, and ${}_3W_1$ and W_{cycle} in terms of P_1, T_1, m, and R.

(c) Sketch the cycle in the $P - v$ plane.

Solution

First, we see that

$$v_1 = \frac{RT_1}{P_1}.$$

Because $1 \to 2$ is isochoric, we have

$$v_2 = v_1 = \frac{RT_1}{P_1}.$$

Also note that

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1},$$

so

$$P_2 = P_1 \frac{v_1}{v_2}\frac{T_2}{T_1} = P_1 \frac{v_1}{v_1}\frac{2T_1}{T_1},$$

$$\boxed{P_2 = 2P_1.}$$

For the isochoric process, we have

$$\boxed{{}_1W_2 = 0.}$$

Now we know that

$$T_3 = T_2 = 2T_1.$$

And we have $P_3 = P_1$, so

$$v_3 = \frac{RT_3}{P_3} = \frac{2RT_1}{P_1} = 2v_1 = 2v_2.$$

Thus $v_3/v_2 = 2$. For the isothermal process, we have

$${}_2W_3 = mRT_2 \ln \frac{v_3}{v_2} = \boxed{2mRT_1 \ln 2}.$$

For the isobaric process from 3 to 1, we have

$${}_3W_1 = mP_1(v_1 - v_3) = mP_1 v_1 \left(1 - \frac{v_3}{v_1}\right) = mRT_1\,(1 - 2) = \boxed{-mRT_1}.$$

Therefore, the total work of the cycle is

$$W_{cycle} = 0 + 2mRT_1 \ln 2 - mRT_1.$$

$$\boxed{W_{cycle} = (2\ln 2 - 1)mRT_1 = 0.386294mRT_1.}$$

Note that $W_{cycle} > 0$.

The appropriate sketch is given next.

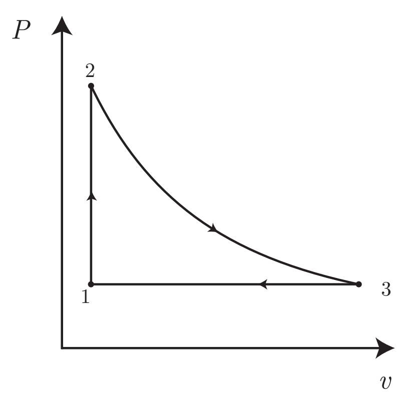


Figure 3: Sketch of cycle in $P - v$ plane.

5. (30) A fixed mass, $m = 10$ kg, of H_2O is initially at $P = 100$ kPa, $x = 0.4$. It undergoes a two-step process. The first step is an isobaric heating until $T = 200°$C. The second step is an isothermal compression until $P = 500$ kPa.

   (a) Find v at the end of the isobaric heating.
   (b) Find the total work in the two-step process.
   (c) Sketch the two-step process, including the vapor domes and saturation lines, in the $T-v$, $P-v$, and $P-T$ planes.

Solution

Table B.1.2, the saturated water pressure entry table helps us with state 1. At $P_1 = 100$ kPa, we find $v_{f1} = 0.001045$ m^3/kg, $v_{fg1} = 1.69296$ m^3/kg. So

$$v_1 = v_{f1} + x_1 v_{fg1} = \left(0.001045\ \frac{\text{m}^3}{\text{kg}}\right) + (0.4)\left(1.69696\ \frac{\text{m}^3}{\text{kg}}\right) = 0.678227\ \frac{\text{m}^3}{\text{kg}}.$$

Now when $T_2 = 200°$C and $P_2 = P_1 = 100$ kPa. The tables reveal state 2 is a superheated vapor with

$$\boxed{v_2 = 2.17226\ \frac{\text{m}^3}{\text{kg}}.}$$

Now the work for the isobaric part of the process is straightforward,

$${}_1W_2 = mP_1(v_2 - v_1) = (10\text{ kg})(100\text{ kPa})\left(\left(2.17226\ \frac{\text{m}^3}{\text{kg}}\right) - \left(0.678227\ \frac{\text{m}^3}{\text{kg}}\right)\right) = 1494.03\text{ kJ}.$$

We need the path in $P-v$ space to get the work for the isothermal compression. The points in the $P-v$ plane on the $T_2 = T_3 = 200°$C isotherm are found in the superheated water vapor tables. We find five points with values and those are

$$\begin{aligned}
P_a &= 100\text{ kPa}, & v_a &= 2.17226\ \frac{\text{m}^3}{\text{kg}},\\
P_b &= 200\text{ kPa}, & v_b &= 1.08034\ \frac{\text{m}^3}{\text{kg}},\\
P_c &= 300\text{ kPa}, & v_c &= 0.71629\ \frac{\text{m}^3}{\text{kg}},\\
P_d &= 400\text{ kPa}, & v_d &= 0.53422\ \frac{\text{m}^3}{\text{kg}},\\
P_e &= 500\text{ kPa}, & v_e &= 0.42492\ \frac{\text{m}^3}{\text{kg}}.
\end{aligned}$$

Here state a is the 2 state and state e is the 3 state. As done in class, we approximate

$${}_2W_3 = m\int_2^3 P\ dv = m\sum P_{ave,i}(v_{i+1} - v_i).$$

Doing this, we get

$$\begin{aligned}
{}_2w_3 &= \left(\frac{P_a + P_b}{2}\right)(v_b - v_a)\\
&\quad + \left(\frac{P_b + P_c}{2}\right)(v_c - v_b)\\
&\quad + \left(\frac{P_c + P_d}{2}\right)(v_d - v_c)\\
&\quad + \left(\frac{P_d + P_e}{2}\right)(v_e - v_d),\\
&= \left(\frac{(100\text{ kPa}) + (200\text{ kPa})}{2}\right)\left(\left(1.08034\ \frac{\text{m}^3}{\text{kg}}\right) - \left(2.17226\ \frac{\text{m}^3}{\text{kg}}\right)\right)\\
&\quad + \left(\frac{(200\text{ kPa}) + (300\text{ kPa})}{2}\right)\left(\left(0.71629\ \frac{\text{m}^3}{\text{kg}}\right) - \left(1.08034\ \frac{\text{m}^3}{\text{kg}}\right)\right)\\
&\quad + \left(\frac{(300\text{ kPa}) + (400\text{ kPa})}{2}\right)\left(\left(0.53422\ \frac{\text{m}^3}{\text{kg}}\right) - \left(0.71629\ \frac{\text{m}^3}{\text{kg}}\right)\right)\\
&\quad + \left(\frac{(400\text{ kPa}) + (500\text{ kPa})}{2}\right)\left(\left(0.42492\ \frac{\text{m}^3}{\text{kg}}\right) - \left(0.53422\ \frac{\text{m}^3}{\text{kg}}\right)\right),\\
&= -367.71\ \frac{\text{kJ}}{\text{kg}}.
\end{aligned}$$

So

$$_2W_3 = m_2 w_3 = (10 \text{ kg}) \left(-367.71 \ \frac{\text{kJ}}{\text{kg}} \right) = -3677.1 \text{ kJ}.$$

And for the total process

$$_1W_3 = {_1W_2} + {_2W_3} = (1494.03 \text{ kJ}) + (-3677.1 \text{ kJ}) = \boxed{-2183.07 \text{ kJ}.}$$

The plots follow.

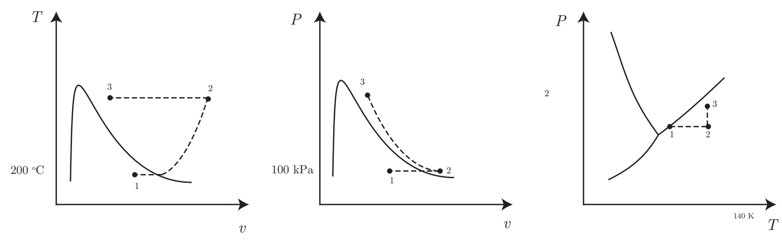


Figure 4: Two-step isobaric, isothermal process in $T-v$, $P-v$, and $P-T$ planes (not to scale).