NAME: SOLUTION
AME 20231, Thermodynamics
Examination 2
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1. (20) A cubical mass of 10 kg of cast iron is at 500 K. It comes to thermal equilibrium with its surroundings, that are at $T_\infty = 300$ K. The convective heat transfer coefficient is $\mathsf{h} = 20$ kW/m^2/K. As done in class, assume the temperature of the cast iron is spatially uniform throughout its volume.

   (a) Find an expression for $T(t)$ of the cast iron cube.

   (b) Evaluate the time constant associated with this cooling process.

Solution

The first law, with no work for the incompressible cast iron, tells us

$$\frac{dU}{dt} = -\mathsf{h}A(T - T_\infty).$$

The mass m and specific heat c are constant, so with $dU = mcdT$, we get

$$mc\frac{dT}{dt} = -\mathsf{h}A(T - T_\infty).$$
$$\frac{dT}{dt} = -\frac{\mathsf{h}A}{mc}(T - T_\infty).$$
$$\frac{dT}{T - T_\infty} = -\frac{\mathsf{h}A}{mc}dt,$$
$$\ln(T - T_\infty) = -\frac{\mathsf{h}A}{mc}t + C.$$
$$T - T_\infty = (T_o - T_\infty)\exp\left(-\frac{\mathsf{h}A}{mc}t\right)$$
$$T(t) = T_\infty + (T_o - T_\infty)\exp\left(-\frac{\mathsf{h}A}{mc}t\right).$$

The time constant is

$$\tau = \frac{mc}{\mathsf{h}A}.$$

Let us put some numbers to this. Say the cube has volume ℓ^3, with ℓ the length of a side. Then the area of one side is ℓ^2 and the total surface area is $6\ell^2$. The mass is

$$m = \rho V = \rho\ell^3$$

. So

$$\ell = \left(\frac{m}{\rho}\right)^{1/3} = \left(\frac{10 \text{ kg}}{7272 \frac{\text{kg}}{\text{m}^3}}\right)^{1/3} = 0.111203 \text{ m}.$$

$$\tau = \frac{\rho\ell^3 c}{\mathsf{h}6\ell^2} = \frac{\rho\ell c}{6\mathsf{h}} = \frac{\left(7272 \frac{\text{kg}}{\text{m}^3}\right) 0.111203 \text{ m}}{6\left(20 \frac{\text{kW}}{\text{m}^2 \text{ K}}\right)} \boxed{= 2.83 \text{ s.}}$$

Thus,

$$T(t) = (300 \text{ K}) + ((500 \text{ K}) - (300 \text{ K}))\exp\left(\frac{-t}{2.83 \text{ s}}\right).$$

$$\boxed{T(t) = (300 \text{ K}) + (200 \text{ K})\exp\left(\frac{-t}{2.83 \text{ s}}\right).}$$

2. (30) A fixed mass m of calorically imperfect ideal gas (CIIG) with gas constant R and specific heat at constant volume

$$c_v = c_{vo} + \alpha T,$$

is initially at $T = T_1$ and specific volume $v = v_1$.

(a) Find an expression for the specific entropy s of the gas as a function of T and v, parameterized by c_{vo}, R, and reference state values T_o and v_o.

(b) The gas *isochorically* comes to thermal equilibrium with its surroundings, which are at $T = T_\infty$. Find the total entropy change of the system, surroundings, and universe.

Solution

We can begin with the Gibbs equation

$$du = T\ ds - P\ dv.$$

Rearrange to get

$$ds = \frac{du}{T} + \frac{P}{T}\ dv.$$

Because $du = c_v(T)\ dT$ and because $Pv = RT$, we can say

$$ds = \frac{(c_{vo} + \alpha T)\ dT}{T} + \frac{R}{v}\ dv.$$

$$\boxed{s - s_o = c_{vo} \ln \frac{T}{T_o} + \alpha(T - T_o) + R \ln \frac{v}{v_o}.}$$

For this process, we have $T_2 = T_\infty$. So we get

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v(T)\ dT,$$

$$u_2 - u_1 = \int_{T_1}^{T_\infty} (c_{vo} + \alpha T)\ dT,$$

$$u_2 - u_1 = c_{vo}(T_\infty - T_1) + \frac{\alpha}{2}(T_\infty^2 - T_1^2).$$

$$u_2 - u_1 = \left(c_{vo} + \alpha \frac{T_\infty + T_1}{2}\right)(T_\infty - T_1).$$

Because the process is isochoric, the heat transfer is the change in internal energy, so

$${}_1q_2 = u_2 - u_1 = \left(c_{vo} + \alpha \frac{T_\infty + T_1}{2}\right)(T_\infty - T_1).$$

And for the total system, we get

$${}_1Q_2 = m(u_2 - u_1) = m\left(c_{vo} + \alpha \frac{T_\infty + T_1}{2}\right)(T_\infty - T_1).$$

For the system, the entropy change is

$$s_2 - s_1 = c_{vo} \ln \frac{T_\infty}{T_1} + \alpha(T_\infty - T_1), \qquad \boxed{S_2 - S_1 = m\left(c_{vo} \ln \frac{T_\infty}{T_1} + \alpha(T_\infty - T_1)\right).}$$

The surroundings has entropy change

$$\boxed{\Delta S_{surr} = -\frac{{}_1Q_2}{T_\infty} = \frac{m\left(c_{vo} + \alpha \frac{T_\infty + T_1}{2}\right)(T_1 - T_\infty)}{T_\infty}.}$$

This gives

$$\Delta S_{univ} = S_2 - S_1 + \Delta S_{surr}.$$

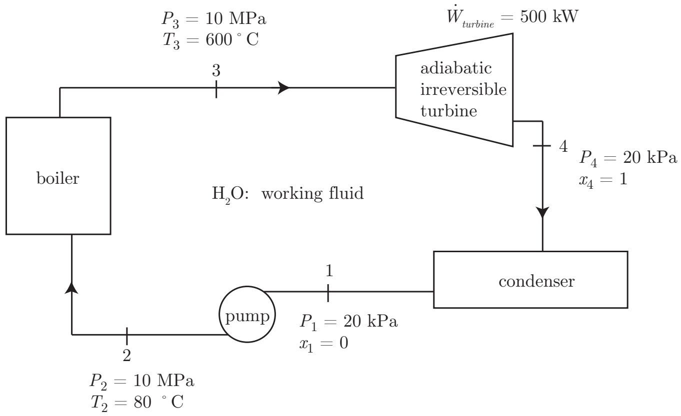
$$\boxed{\Delta S_{univ} = m\left(c_{vo} \ln \frac{T_\infty}{T_1} + \alpha(T_\infty - T_1) + \frac{\left(c_{vo} + \alpha \frac{T_\infty + T_1}{2}\right)(T_1 - T_\infty)}{T_\infty}\right).}$$

Detailed analysis, valid in the limit for which $T_\infty \sim T_1(1 + \epsilon)$, where $\epsilon \to \pm 0$, followed by Taylor series expansion in that limit shows that

$$\lim_{\epsilon \to 0} \Delta S_{univ} = \frac{1}{2} m(c_{vo} + \alpha T_\infty)\epsilon^2 + \ldots.$$

Thus, either positive or negative deviation from T_∞ results in second law satisfaction because ΔS_{univ} depends on the square of the deviation, ϵ^2.

3. (50) Consider the Rankine cycle below. Find



(a) the specific work done by the adiabatic irreversible turbine (kJ/kg),

(b) the mass flow rate (kg/s),

(c) the heat transfer rate to the boiler (kW),

(d) the work rate required to power the pump (kW),

(e) the overall thermal efficiency,

(f) the thermal efficiency of a Carnot cycle operating between the same temperature limits,

(g) an accurate sketch of the cycle on a $T - s$ diagram,

(h) the turbine work rate (kW) that could have been achieved had the adiabatic irreversible turbine been replaced by an adiabatic reversible (isentropic) turbine operating with the same inlet state and between the same pressure limits.

Solution

At state 1, we have two properties. From the tables, we learn

$$h_1 = 251.38\ \frac{\text{kJ}}{\text{kg}}, \qquad s_1 = 0.8319\ \frac{\text{kJ}}{\text{kg K}}, \qquad T_1 = 60.06\ ^\circ C.$$

At state 2, the compressed liquid tables give

$$h_2 = 342.81\ \frac{\text{kJ}}{\text{kg}}, \qquad s_2 = 1.0687\ \frac{\text{kJ}}{\text{kg K}}.$$

After the boiler, we know two properties and find

$$h_3 = 3625.34\ \frac{\text{kJ}}{\text{kg}}, \qquad s_3 = 6.9028\ \frac{\text{kJ}}{\text{kg K}}.$$

After the turbine, we know two properties and find

$$h_4 = 2609.70\ \frac{\text{kJ}}{\text{kg}}, \qquad s_4 = 7.9085\ \frac{\text{kJ}}{\text{kg K}}.$$

So the specific work from the turbine is

$${}_3w_4 = (h_3 - h_4) = \left(\left(3625.34\ \frac{\text{kJ}}{\text{kg}}\right) - \left(2609.70\ \frac{\text{kJ}}{\text{kg}}\right)\right) \boxed{= 1015.64\ \frac{\text{kJ}}{\text{kg}}.}$$

Because $\dot{W}_{turbine} = \dot{m}\,{}_3w_4$, we have

$$\dot{m} = \frac{\dot{W}_{turbine}}{{}_3w_4} = \frac{500\ \text{kW}}{1015.64\ \frac{\text{kJ}}{\text{kg}}} = \boxed{0.4923\ \frac{\text{kg}}{\text{s}}.}$$

For the boiler we have

$$_2\dot{Q}_3 = \dot{m}(h_3 - h_2) = \left(0.4923\ \frac{\text{kg}}{\text{s}}\right)\left(\left(3625.34\ \frac{\text{kJ}}{\text{kg}}\right) - \left(342.81\ \frac{\text{kJ}}{\text{kg}}\right)\right) = \boxed{1615.99\ \text{kW}.}$$

The power requirement of the pump is

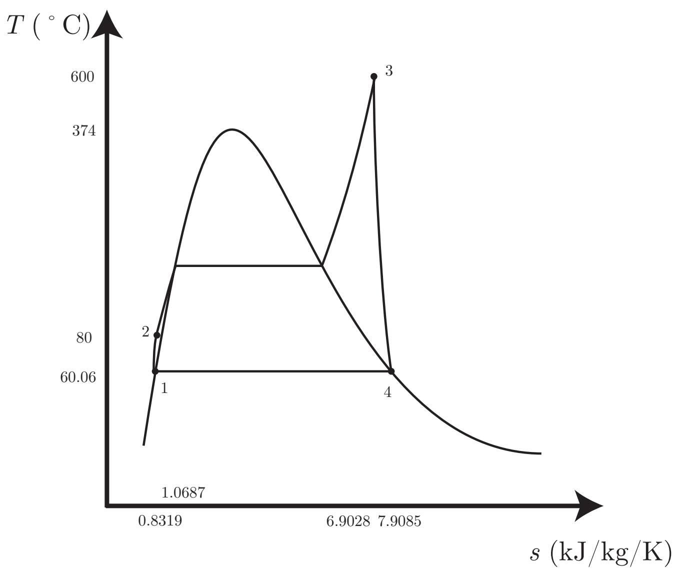
$$_1\dot{W}_2 = \dot{m}(h_2 - h_1) = \left(0.4923\ \frac{\text{kg}}{\text{s}}\right)\left(\left(342.81\ \frac{\text{kJ}}{\text{kg}}\right) - \left(251.38\ \frac{\text{kJ}}{\text{kg}}\right)\right) = \boxed{45.011\ \text{kW}.}$$

The cycle efficiency is

$$\boxed{\eta = \frac{\dot{W}_{net}}{_2\dot{Q}_3} = \frac{(500\ \text{kW}) - (45.011\ \text{kW})}{1615.99\ \text{kW}} = 0.281554.}$$

The Carnot efficiency for an engine operating between the same temperature limits is

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{60.06 + 273.15}{600 + 273.15}\boxed{= 0.618382.}$$



Now if our turbine had been adiabatic reversible, we would have had

$$s_4 = s_3 = 6.9028\ \frac{\text{kJ}}{\text{kg K}}.$$

We still would have dropped to 20 kPa. This would place is under the dome. At this pressure, the tables reveal

$$s_f = 0.8319\ \frac{\text{kJ}}{\text{kg K}}, \qquad s_{fg} = 7.0766\ \frac{\text{kJ}}{\text{kg K}}.$$

$$h_f = 251.38\ \frac{\text{kJ}}{\text{kg}}, \qquad h_{fg} = 2358.33\ \frac{\text{kJ}}{\text{kg}}.$$

Thus

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{\left(6.9028\ \frac{\text{kJ}}{\text{kg K}}\right) - \left(0.8319\ \frac{\text{kJ}}{\text{kg K}}\right)}{7.0766\ \frac{\text{kJ}}{\text{kg K}}} = 0.857884.$$

Thus we find the new h_4 as

$$h_4 = h_f + x_4 h_{fg} = \left(251.38\ \frac{\text{kJ}}{\text{kg}}\right) + 0.989119\left(2358.33\ \frac{\text{kJ}}{\text{kg}}\right) = 2274.55\ \frac{\text{kJ}}{\text{kg}}.$$

Thus the work rate done by the equivalent isentropic turbine is

$$\dot{W}_{isentropic\ turbine} = \left(0.4923\ \frac{\text{kg}}{\text{s}}\right)\left(\left(3625.34\ \frac{\text{kJ}}{\text{kg}}\right) - \left(2274.55\ \frac{\text{kJ}}{\text{kg}}\right)\right) = \boxed{664.993\ \text{kW}.}$$

The adiabatic reversible turbine does more work than the adiabatic irreversible turbine.