NAME: SOLUTION
AME 20231, Thermodynamics
Examination 1
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1. (5) A three phase mixture of H_2O exists at the triple point. The material is isothermally compressed to a pressure just above the triple point pressure. Which phase is observed: solid, liquid, or gas? Provide a sketch of the process in the $P-T$ plane that includes the various phase boundaries and the triple point.

Solution

The figure shows a sketch of the process for water, a material that expands on freezing. For such a material, isothermal compression from the triple point leads to the

$$\boxed{\text{liquid}}$$

phase.

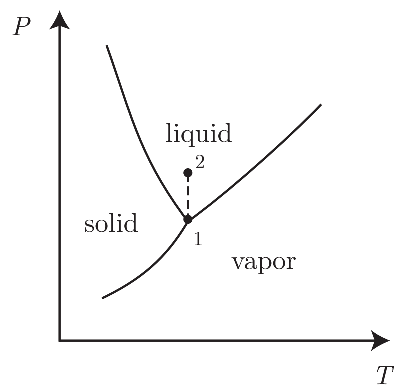


Figure 1: Sketch of process in the $P-T$ plane (not to scale).

2. (15) A box contains one kmole of the the noble gas helium, He, at $T = 6.2$ K, $P = 0.227$ MPa. Determine the volume of the box by two different methods:

   (a) assume an ideal gas,

   (b) use the compressibility chart, Fig. D.1.

Solution

First use the ideal gas law. We must be careful with the units.

$$PV = n\overline{R}T$$

$$V = \frac{n\overline{R}T}{P} = \frac{(1 \text{ kmol})\left(8.314 \frac{\text{kJ}}{\text{kmol K}}\right)(6.2 \text{ K})}{0.227 \times 10^3 \text{ kPa}} = \boxed{0.227 \text{ m}^3}.$$

For the compressibility charts, we need the critical temperature and pressure of Helium. Table A.2 tells us $T_c = 5.19$ K and $P_c = 0.227$ MPa. Thus the reduced temperature and pressure of our system is

$$T_r = \frac{T}{T_c} = \frac{6.2 \text{ K}}{5.19 \text{ K}} = 1.194,$$

$$P_r = \frac{P}{P_c} = \frac{0.227 \text{ MPa}}{0.227 \text{ MPa}} = 1.$$

The Lee-Kessler generalized compressibility chart of Fig. D.1 tells us then that $Z \sim 0.78$. So, we get

$$Z = \frac{PV}{n\overline{R}T},$$

$$V = Z\frac{n\overline{R}T}{P} = (0.78)\frac{(1 \text{ kmol})\left(8.314\frac{\text{kJ}}{\text{kmol K}}\right)(6.2 \text{ K})}{0.227 \times 10^3 \text{ kPa}} = \boxed{0.177 \text{ m}^3}.$$

3. (15) The gas CO_2 exists at $P = 10000$ kPa, $T = 203°$C. Determine the specific volume of the gas by two different methods:

   (a) assume an ideal gas,

   (b) use Table B.3.2.

Solution

First use the ideal gas law. We need the absolute temperature:

$$T = 203 + 273.15 = 476.15 \text{ K}.$$

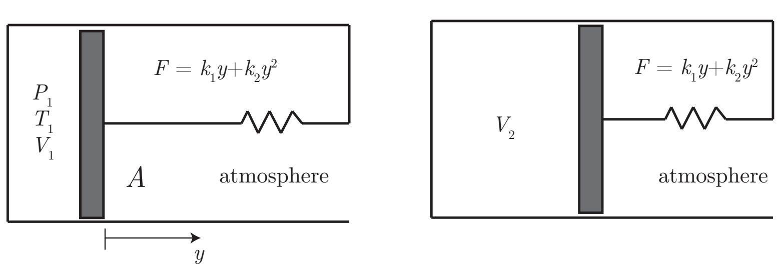
We also need R for CO_2. This is found in Table A.5, giving $R = 0.1889$ kJ/kg/K. The ideal gas law gives

$$v = \frac{RT}{P} = \frac{\left(0.1889\ \frac{\text{kJ}}{\text{kg K}}\right)(476.15 \text{ K})}{10000 \text{ kPa}} = \boxed{0.00899\ \frac{\text{m}^3}{\text{kg}}}.$$

Next use Table B.3.2. We need to interpolate. We use the point slope formula to get

$$v = \left(0.00819\ \frac{\text{m}^3}{\text{kg}}\right) + \frac{\left(0.00868\ \frac{\text{m}^3}{\text{kg}}\right) - \left(0.00819\ \frac{\text{m}^3}{\text{kg}}\right)}{220°\text{C} - 200°\text{C}}(203°\text{C} - 200°\text{C}) = \boxed{0.00826\ \frac{\text{m}^3}{\text{kg}}}.$$

4. (30) A piston-cylinder arrangement contains an ideal gas with gas constant R at initial temperature, pressure, and volume T_1, P_1, V_1. The piston, of cross-sectional area A, is constrained by a nonlinear spring that exerts no force in the initial configuration. The force in the spring is given by the formula $F = k_1 y + k_2 y^2$, where y is the displacement of the spring from its initial unstreched position at $y = 0$. The gas is heated to a final volume V_2. Find



(a) the initial specific volume, v_1,
(b) the mass m of the gas,
(c) the atmospheric pressure,
(d) the final pressure P_2,
(e) the work done in the process ${}_1W_2$,

Solution

The ideal gas law gives

$$P_1 v_1 = RT_1,$$

so

$$v_1 = \boxed{\frac{RT_1}{P_1}}.$$

The mass m is $m = V_1/v_1$, so

$$m = \frac{V_1}{\frac{RT_1}{P_1}} = \boxed{\frac{P_1 V_1}{RT_1}}.$$

We first need to consider the geometry. We must have

$$V = V_1 + Ay.$$

Thus

$$y = \frac{V - V_1}{A}.$$

We need a relationship between P and V. Newton's second law, assuming static equilibrium, gives

$$PA = P_{atm}A + k_1 y + k_2 y^2.$$

We can divide by A to get

$$P = P_{atm} + \frac{k_1}{A} y + \frac{k_2}{A} y^2.$$

At the initial state $y = 0$, and $P = P_1$ so for static equilibrium, we must have

$$\boxed{P_1 = P_{atm}.}$$

Using this, and eliminating y in favor of V, we get

$$P = P_1 + \frac{k_1}{A}\left(\frac{V - V_1}{A}\right) + \frac{k_2}{A}\left(\frac{V - V_1}{A}\right)^2.$$

The final pressure is thus

$$\boxed{P_2 = P_1 + \frac{k_1}{A}\left(\frac{V_2 - V_1}{A}\right) + \frac{k_2}{A}\left(\frac{V_2 - V_1}{A}\right)^2.}$$

We can integrate the relationship between P and V to get the work:

$${}_1W_2 = \int_{V_1}^{V_2} P \, dV.$$

This gives

$${}_1W_2 = \int_{V_1}^{V_2} \left(P_1 + \frac{k_1}{A}\left(\frac{V - V_1}{A}\right) + \frac{k_2}{A}\left(\frac{V - V_1}{A}\right)^2 \right) dV.$$

Performing the integration yields

$$\boxed{{}_1W_2 = P_1(V_2 - V_1) + \frac{k_1}{2A^2}(V_2 - V_1)^2 + \frac{k_2}{3A^3}(V_2 - V_1)^3.}$$

5. (35) A piston-cylinder configuration contains 10 kg of H_2O at an initial state of $P_1 = 10000$ kPa, and quality $x_1 = 0$. It is isothermally compressed to $P_2 = 15000$ kPa. It is then isobarically heated to $T_3 = 600°C$. Find

(a) the intermediate specific volume v_2,

(b) the final specific volume v_3,

(c) the total work done in the process ${}_1W_3$,

(d) sketches of the process in the $P-v$, $T-v$, and $P-T$ planes, taking special care to include relevant vapor domes and saturation lines and the correct orientation of the processes relative to these features.

Solution

From Table B.1.2, we learn $v_1 = v_f = 0.001452$ m^3/kg and $T_1 = 311.06$ °C. State 2 is a compressed liquid with $P_2 = 15000$ kPa, and Table B.1.4 tells us by interpolating to $T_2 = 311.06$ °C that

$$v_2 = \left(0.001377 \ \frac{\text{m}^3}{\text{kg}}\right) + \frac{\left(0.001472 \ \frac{\text{m}^3}{\text{kg}}\right) - \left(0.001377 \ \frac{\text{m}^3}{\text{kg}}\right)}{320^\circ\text{C} - 300^\circ\text{C}} (311.06^\circ\text{C} - 300^\circ\text{C})$$

$$\boxed{v_2 = 0.00142954 \ \frac{\text{m}^3}{\text{kg}}.}$$

For the isobaric heating, we have $P_3 = 15000$ kPa. With $T_3 = 600^\circ$C, the final state is seen to be a superheated vapor, with supercritical temperature $T_3 > T_c = 374.14$ °C, and subcritical pressure, $P_3 < P_c = 22089$ kPa. The superheated tables, Table B.1.3, tells us that

$$\boxed{v_3 = 0.02491 \ \frac{\text{m}^3}{\text{kg}}.}$$

The total work is the sum of the work of the isothermal and isobaric processes:

$${}_1W_3 = {}_1W_2 + {}_2W_3,$$
$${}_1W_3 = m \ {}_1w_2 + m \ {}_2w_3,$$
$${}_1W_3 = m\int_{v_1}^{v_2} P \ dv + m\int_{v_2}^{v_3} P \ dv.$$

The first integral for the isothermal process requires a numerical integration. We only have two points from the tables, so we will simply find the area of a simple trapezoid. The second integral for the isobaric process simplifies because the pressure is constant. Let us then take

$${}_1W_3 = m\left(\frac{P_1 + P_2}{2}(v_2 - v_1)\right) + mP_2(v_3 - v_2).$$

$$\begin{aligned}
{}_1W_3 &= (10 \text{ kg})\left(\frac{10000 \text{ kPa} + 15000 \text{ kPa}}{2}\left(\left(0.00142954 \ \frac{\text{m}^3}{\text{kg}}\right) - \left(0.001452 \ \frac{\text{m}^3}{\text{kg}}\right)\right)\right) \\
&\quad + (10 \text{ kg})(15000 \text{ kPa})\left(\left(0.02491 \ \frac{\text{m}^3}{\text{kg}}\right) - \left(0.00142954 \ \frac{\text{m}^3}{\text{kg}}\right)\right), \\
&= -2.80812 \text{ kJ} + 3522.07 \text{ kJ}.
\end{aligned}$$

Adding, we find

$$\boxed{{}_1W_3 = 3519.26 \text{ kJ}.}$$

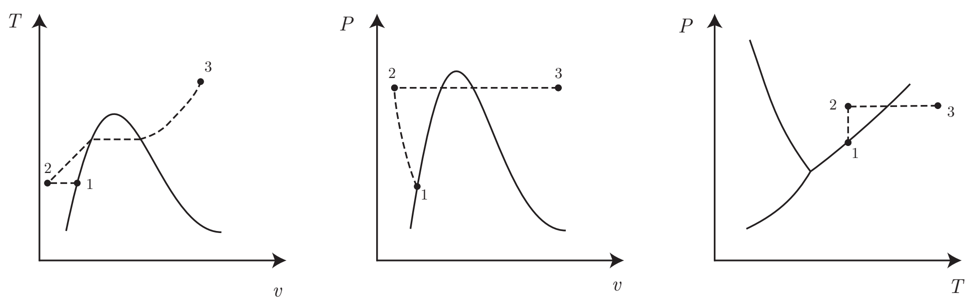


Figure 2: Sketch of process in $T-v$, $P-v$, and $P-T$ planes (not to scale).