NAME: SOLUTION
AME 20231, Thermodynamics
Examination 2
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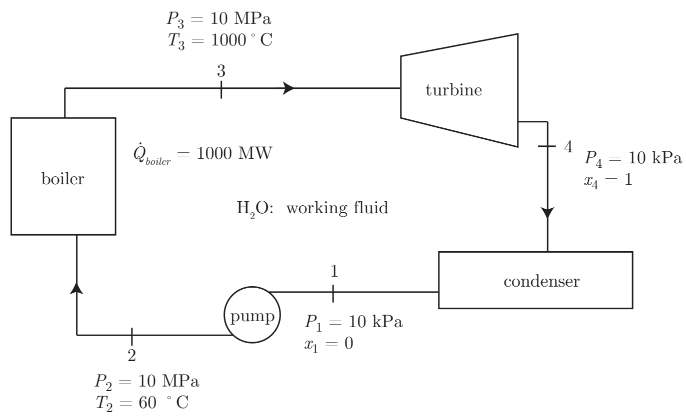
1. (40) Consider the Rankine cycle below. Find



(a) the mass flow rate (kg/s),
(b) the work rate done by the turbine (kW),
(c) the work rate required to power the pump (kW),
(d) the overall thermal efficiency,
(e) a correctly oriented sketch, including the vapor dome and appropriate numerical values of P and v, of the cycle on a $P-v$ diagram,

Solution

The tables give us

$$h_1 = 191.81\ \frac{\text{kJ}}{\text{kg}},\quad h_2 = 259.47\ \frac{\text{kJ}}{\text{kg}},\quad h_3 = 4611.04\ \frac{\text{kJ}}{\text{kg}},\quad h_4 = 2584.63\ \frac{\text{kJ}}{\text{kg}}$$

$$v_1 = 0.001010\ \frac{\text{m}^3}{\text{kg}},\quad v_2 = 0.001013\ \frac{\text{m}^3}{\text{kg}},\quad v_3 = 0.05832\ \frac{\text{m}^3}{\text{kg}},\quad v_4 = 14.67355\ \frac{\text{m}^3}{\text{kg}}$$

For the boiler we have

$${}_2q_3 = h_3 - h_2 = \left(4611.04\ \frac{\text{kJ}}{\text{kg}}\right) - \left(259.47\ \frac{\text{kJ}}{\text{kg}}\right) = 4351.57\ \frac{\text{kJ}}{\text{kg}}$$

So we have

$${}_2\dot{Q}_3 = \dot{m}\ {}_2q_3$$

$$\dot{m} = \frac{{}_2\dot{Q}_3}{{}_2q_3} = \frac{10^6\ \text{kW}}{4351.57\ \frac{\text{kJ}}{\text{kg}}} = \boxed{229.802\ \frac{\text{kg}}{\text{s}}}$$

The specific turbine work is

$${}_3w_4 = h_3 - h_4 = \left(4611.04\ \frac{\text{kJ}}{\text{kg}}\right) - \left(2584.63\ \frac{\text{kJ}}{\text{kg}}\right) = 2026.41\ \frac{\text{kJ}}{\text{kg}}$$

So the turbine power output is

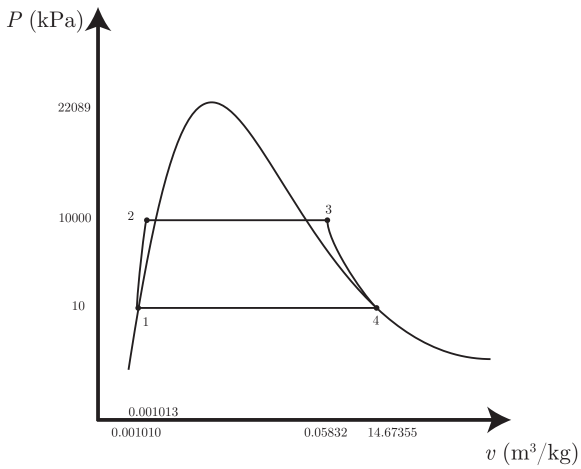
$${}_3\dot{W}_4 = \dot{m}\ {}_3w_4 = \left(229.802\ \frac{\text{kg}}{\text{s}}\right)\left(2026.41\ \frac{\text{kJ}}{\text{kg}}\right) = \boxed{465673\ \text{kW}}.$$

For the pump, we need the following work

$$_1\dot{W}_2 = \dot{m}(h_1 - h_2) = \left(229.802\ \frac{\text{kg}}{\text{s}}\right)\left(191.81\ \frac{\text{kJ}}{\text{kg}}\right) - \left(259.47\ \frac{\text{kJ}}{\text{kg}}\right) = \boxed{-15548.4\ \text{kW}}$$

So the thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{(465673\ \text{kW}) - (15548.4\ \text{kW})}{10^6\ \text{kW}} = \boxed{0.450125}$$



2. (30) A chamber with initial volume $V_1 = 1\ \text{m}^3$ contains air at $P_1 = 100$ kPa, $T_1 = 300$ K. The air is constrained by a piston attached to a *linear spring*. The air is heated to $T_2 = 3000$ K, $P_2 = 200$ kPa. Find the heat transfer $_1Q_2$ assuming air is a

   (a) calorically perfect ideal gas, (use Table A.5),

   (b) calorically imperfect ideal gas (use Table A.7.1).

   (c) Give a one-sentence, qualitative, physics-based interpretation as to why one estimate is different than the other.

Solution

The ideal gas law gives us

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1}$$
$$V_2 = \frac{P_1}{P_2}\frac{T_2}{T_1}V_1$$
$$V_2 = \frac{100\ \text{kPa}}{200\ \text{kPa}}\frac{3000\ \text{K}}{300\ \text{K}}(1\ \text{m}^3).$$
$$V_2 = 5\ \text{m}^3.$$

Now we know that for a linear spring $_1W_2 = \int_1^2 P\ dV$ gives us the area of a trapezoid, which is

$$_1W_2 = \frac{P_1 + P_2}{2}(V_2 - V_1)$$
$$_1W_2 = \frac{100\ \text{kPa} + 200\ \text{kPa}}{2}(5\ \text{m}^3 - 1\ \text{m}^3)$$
$$_1W_2 = 600\ \text{kJ}$$

The first law then gives us

$$U_2 - U_1 = {_1Q_2} - {_1W_2}$$

$$
\begin{aligned}
{}_1Q_2 &= U_2 - U_1 + {}_1W_2 \\
{}_1Q_2 &= m(u_2 - u_1) + {}_1W_2
\end{aligned}
$$

Now, we have

$$
\begin{aligned}
m &= \frac{P_1 V_1}{R T_1} \\
m &= \frac{(100 \text{ kPa})(1 \text{ m}^3)}{\left(0.287 \frac{\text{kJ}}{\text{kg K}}\right)(300 \text{ K})} \\
m &= 1.16144 \text{ kg}
\end{aligned}
$$

For a CPIG, we have $u_2 - u_1 = c_v(T_2 - T_1)$, and we take $c_v = 0.717$ kJ/kg/K, so

$$
\begin{aligned}
{}_1Q_2 &= m c_v (T_2 - T_1) + {}_1W_2 \\
{}_1Q_2 &= (1.16144 \text{ kg})\left(0.717 \frac{\text{kJ}}{\text{kg K}}\right)((3000 \text{ K}) - (300 \text{ K})) + 600 \text{ kJ} \\
&\boxed{{}_1Q_2 = 2848.43 \text{ kJ}}
\end{aligned}
$$

For the CIIG, we have from the tables $u_2 = 2664.27$ kJ/kg, $u_1 = 214.36$ kJ/kg. So

$$
{}_1Q_2 = (1.16144 \text{ kg})\left(\left(2664.27 \frac{\text{kJ}}{\text{kg}}\right) - \left(214.36 \frac{\text{kJ}}{\text{kg}}\right)\right) + 600 \text{ kJ}
$$

$$
\boxed{{}_1Q_2 = 3445.54 \text{ kJ}}
$$

For diatomic molecules, more heat is needed because some energy goes to vibrational and rotational modes.

3. (30) A 1 kg block of silver and a 1 kg block of gold are within in a closed, thermally insulated chamber. The silver has initial temperature $T_S(0) = 1000$ K, and the gold has initial temperature $T_G(0) = 300$ K. The two blocks come to a thermal equilibrium so that they have same final temperature.

   (a) Find the equilibrium temperature.

   (b) Taking as a crude model for the heat transfer rate *from* silver to gold

   $$
   \dot{Q} = \left(0.001 \frac{\text{kW}}{\text{K}}\right)(T_S - T_G),
   $$

   find the time constant of equilibration.

Solution

For this problem, there is no work, so $\dot{W} = 0$. For the silver and gold blocks, we have from the first law

$$
\frac{dU_S}{dt} = -\dot{Q}, \qquad \frac{dU_G}{dt} = \dot{Q}
$$

Adding the two we see

$$
\frac{d}{dt}(U_S + U_G) = 0
$$

That is to say the thermal energy of the combined system is conserved. So

$$
U_S + U_G = (U_S + U_G)|_{t=0}
$$

Using the relation for a solid with constant specific heat that U is proportional to mcT, we can say

$$
m_S c_S T_S + m_G c_G T_G = m_S c_S T_{So} + m_G c_G T_{Go}
$$

For us $m_S = m_G = m$, so we could say

$$
c_S T_S + c_G T_G = c_S T_{So} + c_G T_{Go}
$$

So at a general time, we could say

$$T_G(t) = \frac{c_S(T_{So} - T_S(t)) + c_G T_{Go}}{c_G}$$

$$T_G(t) = \frac{c_S}{c_G}(T_{So} - T_S(t)) + T_{Go}$$

At the equilibrium state, $T_S = T_G = T_E$, so

$$c_S T_E + c_G T_E = c_S T_{So} + c_G T_{Go}$$

$$T_E = \frac{c_S T_{So} + c_G T_{Go}}{c_S + c_G}$$

$$T_E = \frac{\left(0.24\ \frac{\text{kJ}}{\text{kg K}}\right)(1000\ \text{K}) + \left(0.13\ \frac{\text{kJ}}{\text{kg K}}\right)(300\ \text{K})}{\left(0.24\ \frac{\text{kJ}}{\text{kg K}}\right) + \left(0.13\ \frac{\text{kJ}}{\text{kg K}}\right)}$$

$$\boxed{T_E = 754.054\ \text{K}}$$

We can rewrite the first law for silver as

$$m c_S \frac{dT_S}{dt} = -\mathtt{h}(T_S - T_G)$$

$$m c_S \frac{dT_S}{dt} = -\mathtt{h}\left(T_S - \left(\frac{c_S}{c_G}(T_{So} - T_S) + T_{Go}\right)\right)$$

$$m c_S \frac{dT_S}{dt} = -\mathtt{h}\left(T_S\left(1 + \frac{c_S}{c_G}\right) - \left(\frac{c_S}{c_G}T_{So} + T_{Go}\right)\right)$$

$$\frac{dT_S}{dt} = -\frac{\mathtt{h}}{m c_S}\left(T_S\left(1 + \frac{c_S}{c_G}\right) - \left(\frac{c_S}{c_G}T_{So} + T_{Go}\right)\right)$$

By inspection, the time constant is

$$\tau = \frac{m c_S}{\mathtt{h}\left(1 + \frac{c_S}{c_G}\right)} = \frac{(1\ \text{kg})\left(0.24\ \frac{\text{kJ}}{\text{kg K}}\right)}{\left(0.001\ \frac{\text{kW}}{\text{K}}\right)\left(1 + \frac{0.24\ \frac{\text{kJ}}{\text{kg K}}}{0.13\ \frac{\text{kJ}}{\text{kg K}}}\right)} = \boxed{84.32\ \text{s}}$$

As an aside, we can divide top and bottom by c_S to rewrite τ as

$$\tau = \frac{m}{\mathtt{h}\left(\frac{1}{c_S} + \frac{1}{c_G}\right)}$$

Defining c_{hm} as the *harmonic mean specific heat*:

$$c_{hm} = \frac{2}{\frac{1}{c_S} + \frac{1}{c_G}}$$

we could say

$$\tau = \frac{m c_{hm}}{2\mathtt{h}}$$

One can solve for the differential equations and get

$$T_S(t) = (754.054\ \text{K}) + (245.946\ \text{K})\exp(-t/(84.32\ \text{s}))$$

$$T_G(t) = (754.054\ \text{K}) - (454.054\ \text{K})\exp(-t/(84.32\ \text{s}))$$