**NAME: SOLUTION**
AME 20231, Thermodynamics
Examination 1
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10 February 2022

1. (10) $H_2O$ has $x = 0.5$, $v = 0.003568$ m$^3$/kg. Find $P$ and $T$. Give an accurate sketch of its location in the $P - v$, $T - v$, and $P - T$ planes, including the vapor dome.

*Solution*

Table B.1.1 gives at 370°C, which is a guess based on looking at Table B.1.1.

$$x = \frac{v - v_f}{v_{fg}} = \frac{0.003568\ \frac{\text{m}^3}{\text{kg}} - 0.002213\ \frac{\text{m}^3}{\text{kg}}}{0.00271\ \frac{\text{m}^3}{\text{kg}}} = 0.5.$$

This is exactly correct! So

$$\boxed{T = 370^\circ\text{C}} \qquad \boxed{P = 21208 \text{ kPa.}}$$

It is very close to the critical point. This was a good guess that turned out exactly right. Trial and error would lead to the same answer. Figure 1 shows location of the point in the various planes.

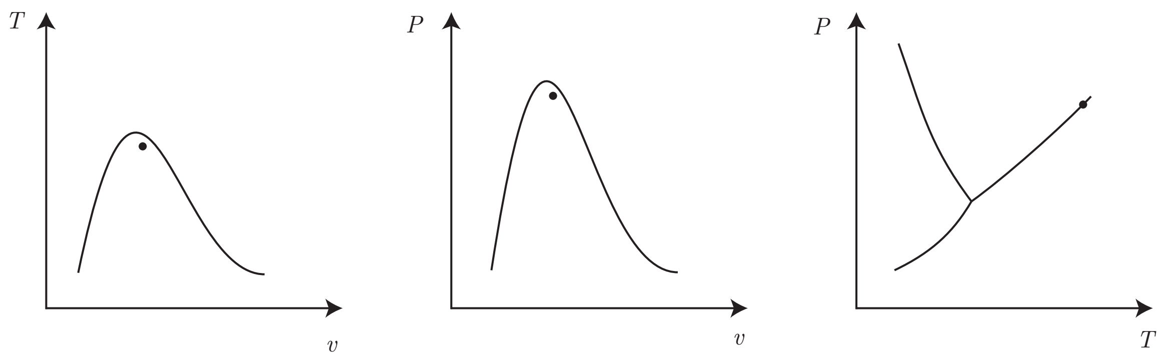


Figure 1: Location of point in the $T - v$, $P - v$, and $P - T$ planes (not to scale).

2. (10) $H_2O$ has $P = 150$ kPa, $T = 580$°C. Find $v$. Give an accurate sketch of its location in the $P - v$, $T - v$, and $P - T$ planes, including the vapor dome.

*Solution*

At 100 kPa, the tables give

$$v = 3.56547\ \frac{\text{m}^3}{\text{kg}} + 0.8\left(4.02781\ \frac{\text{m}^3}{\text{kg}} - 3.5647\ \frac{\text{m}^3}{\text{kg}}\right) = 3.93596\ \frac{\text{m}^3}{\text{kg}}.$$

At 200 kPa, the tables give

$$v = 1.78139\ \frac{\text{m}^3}{\text{kg}} + 0.8\left(2.01297\ \frac{\text{m}^3}{\text{kg}} - 1.78139\ \frac{\text{m}^3}{\text{kg}}\right) = 1.96665\ \frac{\text{m}^3}{\text{kg}}.$$

So at 150 kPa, which is halfway between, we get

$$v = (1/2)\left(3.93596\ \frac{\text{m}^3}{\text{kg}} + 1.96665\ \frac{\text{m}^3}{\text{kg}}\right) = \boxed{2.95131\ \frac{\text{m}^3}{\text{kg}}}.$$

Figure 2 shows location of the point in the various planes.

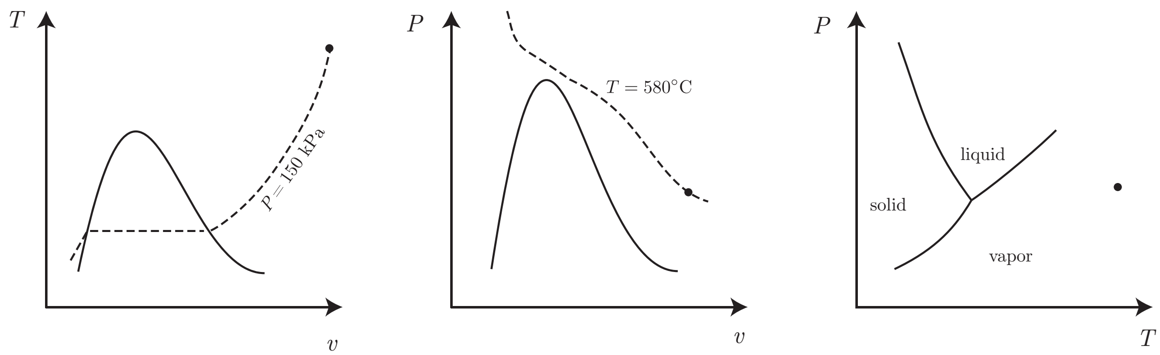


Figure 2: Location of point in the $T-v$, $P-v$, and $P-T$ planes (not to scale).

3. (10) Give an accurate estimate of the gauge and absolute pressure at the bottom of St. Mary's Lake on the Notre Dame campus at its deepest point on an ordinary day. You will need to estimate some geometric parameters for St. Mary's Lake; reasonable estimates are sought.

*Solution*

Take $\rho = 1000$ kg/m$^3$, $g = 9.81$ m/s$^2$, $h = 10$ m, $P_o = 10^5$ Pa. Then

$$P_{gauge} = \rho g h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(10 \text{ m}) = \boxed{98100 \text{ Pa.}}$$

$$P_{abs} = P_o + \rho g h = 10^5 \text{ Pa} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(10 \text{ m}) = \boxed{198100 \text{ Pa.}}$$

4. (35) $H_2O$ has state 1 at the triple point and is all liquid. It undergoes an isochoric process that takes it to $P_2 = 5000$ kPa. It undergoes a polytropic expansion with $n = 1$ that takes it to the triple point pressure at state 3. It then returns isobarically to its original state at the triple point. Find $T_1$, $T_2$, $T_3$ and the net work per unit mass of the cycle. Give an accurate sketch of the process in the $P-v$, $T-v$, and $P-T$ planes. Include the vapor dome.

*Solution*

We have $P_1 = 0.6113$ kPa. And we have

$$\boxed{T_1 = 0.01°\text{C.}}$$

Also $v_1 = 0.001$ m$^3$/kg.

We have $P_2 = 5000$ kPa, $v_2 = 0.001$ m$^3$/kg. So it is now a compressed liquid. The compressed liquid tables tell us that

$$\boxed{T_2 = 20°\text{C}}.$$

For the polytropic process with $n = 1$, we have

$$P_3 v_3 = P_2 v_2.$$

And we are told $P_3 = P_1 = 0.6113$ kPa. So

$$v_3 = \frac{P_2 v_2}{P_3} = \frac{(5000 \text{ kPa})\left(0.001 \frac{\text{m}^3}{\text{kg}}\right)}{0.6113 \text{ kPa}} = 8.17929 \frac{\text{m}^3}{\text{kg}}.$$

This is under the vapor dome and we find

$$x_3 = \frac{v_3 - v_f}{v_{fg}} = \frac{8.17929 \ \frac{\text{m}^3}{\text{kg}} - 0.001 \ \frac{\text{m}^3}{\text{kg}}}{206.131 \ \frac{\text{m}^3}{\text{kg}}} = 0.0396752.$$

And

$$\boxed{T_3 = 0.01^\circ\text{C}.}$$

There is no work from 1 to 2. The net work is

$$w_{net} = {}_2w_3 + {}_3w_1$$

$$w_{net} = P_2 v_2 \int_2^3 \frac{dv}{v} + P_3(v_1 - v_3).$$

$$w_{net} = P_2 v_2 \ln \frac{v_3}{v_2} + P_3(v_1 - v_3).$$

$$w_{net} = (5000 \text{ kPa}) \left(0.001 \ \frac{\text{m}^3}{\text{kg}}\right) \ln \frac{8.17929 \ \frac{\text{m}^3}{\text{kg}}}{0.001 \ \frac{\text{m}^3}{\text{kg}}} + (0.6113 \text{ kPa}) \left(0.001 \ \frac{\text{m}^3}{\text{kg}} - 8.17929 \ \frac{\text{m}^3}{\text{kg}}\right).$$

$$w_{net} = 45.0468 \ \frac{\text{kJ}}{\text{kg}} - 4.99939 \ \frac{\text{kJ}}{\text{kg}} = \boxed{40.0474 \ \frac{\text{kJ}}{\text{kg}}.}$$

Figure 3 shows process in the various planes.

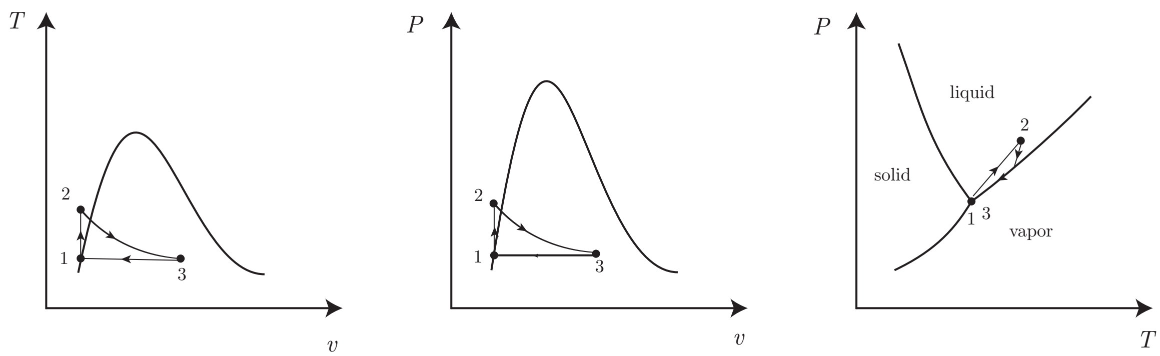


Figure 3: Process for $H_2O$ in the $T - v$, $P - v$, and $P - T$ planes (not to scale).

5. (35) $N_2$ is at $T_1 = 100$ K, $P_1 = 100$ kPa. It is isothermally compressed to $P_2 = 600$ kPa. It is then isochorically compressed to $P_3 = 3000$ kPa. a) Assuming an ideal gas, find $T_3$ and ${}_1w_3$. b) Assuming a non-ideal gas and using Table B.6.2, find $T_3$ and ${}_1w_3$. c) Give an accurate sketch of the process in the $P - v$, $T - v$, and $P - T$ planes. Include the vapor dome.

*Solution*

First assume an ideal gas. We have $R = 0.2968$ kJ/kg/K. We have

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(0.2968 \ \frac{\text{kJ}}{\text{kg K}}\right)(100 \text{ K})}{100 \text{ kPa}} = 0.2968 \ \frac{\text{m}^3}{\text{kg}}.$$

Now

$$v_2 = \frac{P_1 v_1}{P_2} = \frac{(100 \text{ kPa})\left(0.2968 \ \frac{\text{m}^3}{\text{kg}}\right)}{600 \text{ kPa}} = 0.0494667 \ \frac{\text{m}^3}{\text{kg}}.$$

We have

$$v_3 = v_2 = 0.0494667 \ \frac{\text{m}^3}{\text{kg}}.$$

Now

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2},$$

so

$$T_3 = T_2 \frac{P_3 v_3}{P_2 v_2} = T_2 \frac{P_3}{P_2} = (100 \text{ K}) \frac{3000 \text{ kPa}}{600 \text{ kPa}} = \boxed{500 \text{ K.}}$$

The only work is in the isothermal compression so

$$_1w_3 = {_1w_2} = RT_1 \ln \frac{v_2}{v_1} = RT_1 \ln \frac{P_1}{P_2} = \left(0.2968 \frac{\text{kJ}}{\text{kg K}}\right) (100 \text{ K}) \ln \frac{100 \text{ kPa}}{600 \text{ kPa}} = \boxed{-53.1794 \frac{\text{kJ}}{\text{kg}}.}$$

Now assume a non-ideal gas. Note the critical temperature is $T_c = 126.2$ K and critical pressure is $P_c = 3397.8$ kPa. Table B.6.2 gives

$$v_1 = 0.29013 \frac{\text{m}^3}{\text{kg}}.$$

On an isotherm we have

$$\begin{array}{ll} 100 \text{ kPa}, & 0.29013 \dfrac{\text{m}^3}{\text{kg}} \\ 200 \text{ kPa}, & 0.14252 \dfrac{\text{m}^3}{\text{kg}} \\ 400 \text{ kPa}, & 0.06806 \dfrac{\text{m}^3}{\text{kg}} \\ 600 \text{ kPa}, & 0.04299 \dfrac{\text{m}^3}{\text{kg}} \end{array}$$

So $v_2 = 0.04299$ m$^3$/kg. Now $v_3 = v_2 = 0.04299$ m$^3$/kg. With $P_3 = 3000$ kPa, we have

$$\boxed{T_3 \approx 430 \text{ K.}}$$

For the work we take

$$_1w_2 \approx \sum P_{ave} \Delta v = 150(0.14242 - 0.29013) + 300(0.06806 - 0.14252) + 500(0.04299 - 0.06806) = \boxed{-57.0295 \text{ kJ/kg.}}$$

Figure 4 shows process in the various planes.

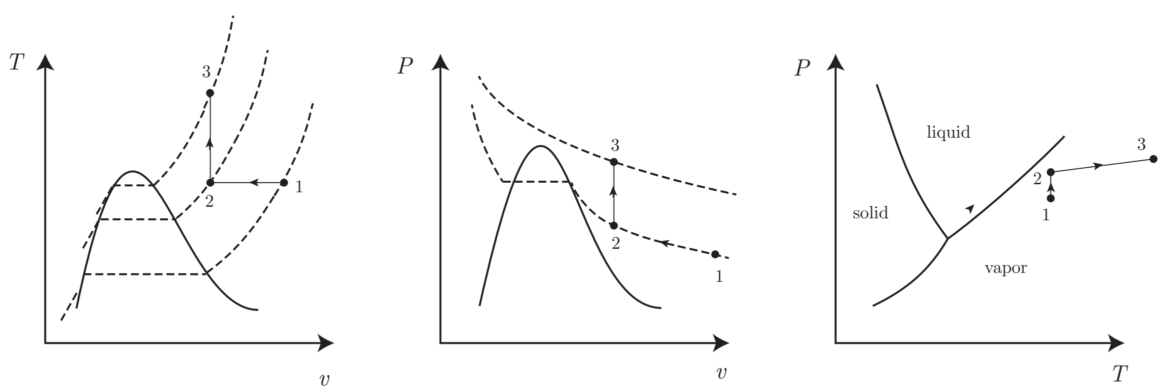


Figure 4: Process for $N_2$ in the $T - v$, $P - v$, and $P - T$ planes (not to scale).