AME 20231, Thermodynamics
Examination 2
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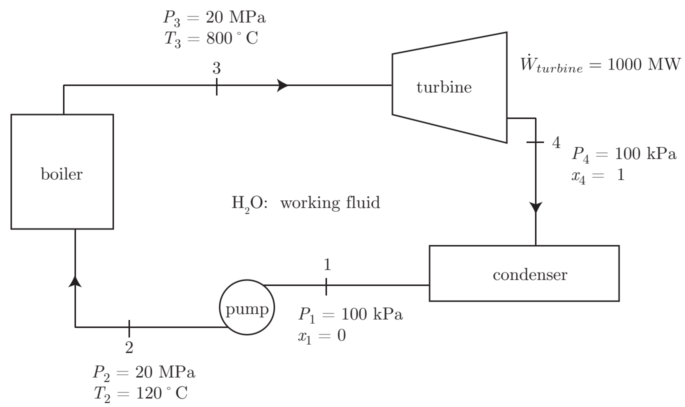
1. (40) Consider the Rankine cycle below. Find



(a) the mass flow rate (kg/s),
(b) the heat addition rate required by the boiler (kW),
(c) the work rate required to power the pump (kW),
(d) the thermal efficiency,
(e) the thermal efficiency of a Carnot cycle operating between the same temperature limits,
(f) a correctly oriented sketch, including the vapor dome and appropriate numerical values of P and v, of the cycle on a $P-v$ diagram,

Solution

Most students did very well on this. A small number were very confused and will need to study this problem very carefully as good performance is expected on problems of this type. Some of the plots were off. The pressure is below the critical pressure. State 3 is not on the vapor dome; it is a superheated state. Process 2 to 3 is isobaric.

The tables give us

$$h_1 = 417.44\ \frac{\text{kJ}}{\text{kg}},\quad h_2 = 517.74\ \frac{\text{kJ}}{\text{kg}},\quad h_3 = 4069.80\ \frac{\text{kJ}}{\text{kg}},\quad h_4 = 2675.46\ \frac{\text{kJ}}{\text{kg}}$$

$$v_1 = 0.001043\ \frac{\text{m}^3}{\text{kg}},\quad v_2 = 0.001050\ \frac{\text{m}^3}{\text{kg}},\quad v_3 = 0.02385\ \frac{\text{m}^3}{\text{kg}},\quad v_4 = 1.694\ \frac{\text{m}^3}{\text{kg}}$$

For the boiler we have

$${}_2q_3 = h_3 - h_2 = \left(4069.80\ \frac{\text{kJ}}{\text{kg}}\right) - \left(517.74\ \frac{\text{kJ}}{\text{kg}}\right) = 3552.06\ \frac{\text{kJ}}{\text{kg}}$$

The specific turbine work is

$${}_3w_4 = h_3 - h_4 = \left(4069.80\ \frac{\text{kJ}}{\text{kg}}\right) - \left(2675.46\ \frac{\text{kJ}}{\text{kg}}\right) = 1394.34\ \frac{\text{kJ}}{\text{kg}}$$

So we have

$${}_3\dot{W}_4 = \dot{m}\ {}_3w_4$$

$$\dot{m} = \frac{{}_3\dot{W}_4}{{}_3w_4} = \frac{10^6\ \text{kW}}{1394.34\ \frac{\text{kJ}}{\text{kg}}} = \boxed{717.185\ \frac{\text{kg}}{\text{s}}}$$

For the pump, we need the following work

$$ {}_1\dot{W}_2 = \dot{m}(h_1 - h_2) = \left(717.185\ \frac{\text{kg}}{\text{s}}\right)\left(\left(417.44\ \frac{\text{kJ}}{\text{kg}}\right) - \left(517.74\ \frac{\text{kJ}}{\text{kg}}\right)\right) = \boxed{-71933.7\ \text{kW}} $$

For the boiler then

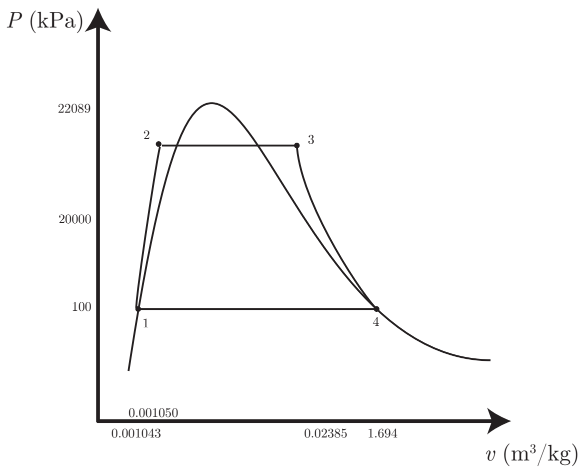
$$ {}_2\dot{Q}_3 = \dot{m}(h_3 - h_2) = \left(717.185\ \frac{\text{kg}}{\text{s}}\right)\left(\left(4069.80\ \frac{\text{kJ}}{\text{kg}}\right) - \left(517.74\ \frac{\text{kJ}}{\text{kg}}\right)\right) = \boxed{2.54748 \times 10^6\ \text{kW}}. $$

So the thermal efficiency is

$$ \eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{(10^6\ \text{kW}) - (71933.7\ \text{kW})}{2.54748 \times 10^6\ \text{kW}} = \boxed{0.364307} $$

For a Carnot engine, we would have

$$ \eta = 1 - \frac{T_4}{T_3} = 1 - \frac{99.62 + 273.15}{800 + 273.15} = \boxed{0.652639}. $$



2. (15) The gas N_2 is at $P_1 = 100$ kPa and $T_1 = 200$ K. It is heated isobarically to $T_2 = 300$ K. Estimate the thermal energy per unit mass required via three different assumptions:

   (a) calorically perfect ideal gas, (use Table A.5),

   (b) calorically imperfect ideal gas (use Table A.8).

   (c) non-ideal gas, (use Table B.6.2).

Solution

This problem was not difficult but many students struggled with it. The biggest problem was approximating ${}_1q_2 = \Delta u$. That is only true for isochoric processes. For the isobaric process, ${}_1q_2 = \Delta h$.

For all problems, the required thermal energy is the change in enthalpy for the isobaric process. For the calorically perfect assumption, we have $c_P = 1.042$ kJ/kg/K. So

$$ {}_1q_2 = \Delta h = c_P(T_2 - T_1) = 1.042\ \frac{\text{kJ}}{\text{kg K}}(100\ \text{K}) = \boxed{104.2\ \frac{\text{kJ}}{\text{kg}}}. $$

For a calorically imperfect ideal gas assumption, we have

$$ {}_1q_2 = \Delta h = \left(311.67\ \frac{\text{kJ}}{\text{kg}}\right) - \left(207.75\ \frac{\text{kJ}}{\text{kg}}\right) = \boxed{103.92\ \frac{\text{kJ}}{\text{kg}}}. $$

For the non-ideal gas assumption we have

$$ {}_1q_2 = \Delta h = \left(311.16\ \frac{\text{kJ}}{\text{kg}}\right) - \left(206.97\ \frac{\text{kJ}}{\text{kg}}\right) = \boxed{104.19\ \frac{\text{kJ}}{\text{kg}}}. $$

3. (30) A 1 kg sphere of gold is initially at $T(0) = 400$ K. It is in an environment with temperature $T_\infty = 300$ K. As done in class, one can approximate the temperature of the sphere as uniform throughout. The thermal energy flux from the sphere to the environment is well modeled by $\dot{Q} = -\mathsf{h}A(T - T_\infty)$, where $\mathsf{h} = 0.01$ kW/m^2/K is the convective heat transfer coefficient, A is the surface area, and T is the time–dependent temperature of the gold. Find $T(t)$ and the time constant associated with the process. Find the total heat transferred to the environment Q.

Solution

Performance was mixed on this. Some students tried to recall memorized equations for $T(t)$ and got mixed up. Much better was to derive the equation for $T(t)$ from the first law of thermodynamics. Calculation of total heat transfer was much easier than many students believed. This was a case where $Q = mc\Delta T$ is exactly correct and easy to apply.

The solution and solution procedure is identical to that found in the class notes, which has a full derivation. The solution is

$$T(t) = T_\infty + (T_o - T_\infty) \exp\left(-\frac{\mathsf{h}A}{\rho c V} t\right).$$

We have $\rho = 19300$ kg/m^3. We get

$$V = \frac{m}{\rho} = \frac{1 \text{ kg}}{19300 \frac{\text{kg}}{\text{m}^3}} = 0.0000518135 \text{ m}^3.$$

So

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = 0.0231269 \text{ m}.$$

And

$$A = 4\pi r^2 = 0.00672119 \text{ m}^2.$$

The solution for $T(t)$ is

$$\boxed{T(t) = (300 \text{ K}) + (100 \text{ K})\, e^{-(0.000517014 \text{ s}^{-1})t}.}$$

The time constant is

$$\tau = \frac{1}{0.000517014 \text{ s}^{-1}} = \boxed{1934.18 \text{ s}}.$$

The total heat transferred the environment is

$$Q = mc\Delta T = (1 \text{ kg})\left(0.13 \frac{\text{kJ}}{\text{kg K}}\right)(100 \text{ K}) = \boxed{13 \text{ kJ}}.$$

4. (15) A Carnot freezer with desired interior temperature of 0°F is in a garage with temperature of 50°F. The freezer draws 300 W of electrical power from a wall outlet. Find the rate of thermal energy released to the garage from the freezer.

Solution

There are many ways to approach this problem. One is given here. Many students got mixed up on units. One has to use absolute temperature units, either Kelvin or Rankine.

We have the first law for a Carnot cycle.

$$\dot{Q}_L + \dot{W} = \dot{Q}_H.$$

For a Carnot cycle, we also have

$$\frac{\dot{Q}_L}{\dot{Q}_H} = \frac{T_L}{T_H}.$$

So

$$\dot{Q}_L = \dot{Q}_H \frac{T_L}{T_H}.$$

Thus the first law is

$$\dot{Q}_H \frac{T_L}{T_H} + \dot{W} = \dot{Q}_H,$$

$$W = \dot{Q}_H \left(1 - \frac{T_L}{T_H}\right),$$

$$\dot{Q}_H = \frac{\dot{W}}{1 - \frac{T_L}{T_H}},$$

$$\dot{Q}_H = \frac{300 \text{ W}}{1 - \frac{0+459.67}{50+459.67}} = \boxed{3058.02 \text{ W.}}$$