AE 360
Design/Numerical Project
J. M. Powers
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This project will be in two somewhat disjoint parts. In the first part, to gain an appreciation of what goes into large scale numerical codes, you will write your own numerical code (the computer language is your choice) to solve a well known one-dimensional shock tube problem. In the second part you will use a similar two-dimensional code to design an airfoil which will operate under supersonic conditions.

Shock Tube Problem

Consider a duct with length 10 m and cross sectional area 0.01 m^2. At $t = 0$ s, air is at rest in the tube at $T = 300$ K. A thin diaphragm at the tube's midplane separates the air into two chambers. On one side the air is at 2000 kPa; the other side is at 100 kPa. At $t = 0+$, the diaphragm is burst, and motion of the air ensues.

You are to write a finite difference code, using a Lax-Wendroff technique, which models this process. See the following page for a description of the Lax-Wendroff technique. Also note the useful diagnostic tool of mass and energy checks described on the same page.

Supersonic Airfoil

Consider an ambient atmosphere at 100 kPa and 300 K. Design an airfoil which can provide sufficient lift to keep a 2000 kg plane airborne at a design Mach number of $M = 2.2$. For this section, I will provide you with an existing Fortran 77 research code which can address this problem.

Deliverables:

- Thursday 3 April 1997
  - Two page maximum LaTeXgenerated technical memorandum
  - Brief written problem description: shock tube only
  - Plots of $P(x,t), \rho(x,t), T(x,t)$
  - Hard copy of your code as an appendix
- Thursday 17 April 1997
  - LaTeXgenerated technical memorandum, 5-10 pages
  - Brief written problem description
  - Your recommended design
  - Description of your analysis
  - Pressure, density, and temperature contours at the design Mach number
  - Prediction of C_D and C_L
  - Discussion of your results
  - Shock tube problem as an appendix

In your reports, check carefully for spelling and grammar, fully define your problem so that your results are *repeatable*. All figures must be computer generated and automatically inserted into your document.

Lax-Wendroff Technique

Consider a system of the form

$$\frac{\partial q}{\partial t} + \frac{\partial f(q)}{\partial x} = 0.$$

Here q and f vector functions of length N; further f is itself a function of q. We discretize so that

$$q(x,t) \to q_i^n$$

$$f(q(x,t)) \to f(q_i^n)$$

The two-step Lax-Wendroff discretization is as follows

- at a given time step estimate q at the $i+1/2$ cell interface:

$$q_{i+1/2}^n = \frac{1}{2}\left(q_i^n + q_{i+1}^n\right).$$

- use central differencing (about $i+1/2$) to step forward $\Delta t/2$ so that $q_{i+1/2}^{n+1/2}$ can be estimated:

$$q_{i+1/2}^{n+1/2} = q_{i+1/2}^n - \frac{\Delta t/2}{\Delta x}\left[f(q_{i+1}^n) - f(q_i^n)\right].$$

- use central differencing (about i) to step forward Δt, evaluating f at the $i \pm 1/2$ and $n+1/2$ steps:

$$q_i^{n+1} = q_i^n - \frac{\Delta t}{\Delta x}\left[f(q_{i+1/2}^{n+1/2}) - f(q_{i-1/2}^{n+1/2})\right].$$

Check for global mass and energy conservation

It is easily shown that for global mass and energy conservation one must have *at each step in time*:

$$\sum_{i=1}^{N} \rho_i = K_1, \qquad \sum_{i=1}^{N} \rho_i \left(e_i + u_i^2/2\right) = K_2.$$

Here K_1 and K_2 are constants should remain essentially constant throughout the calculation. Any changes will be due solely to roundoff error and confined to the last one or two digits at most. Inserting a check calculation of this and printing the result to the screen at each time step is a very useful diagnostic.