TO: Joseph M. Powers
FROM: Brian P.Rigney
DATE: 3 April 1997
RE: AE 360 Project: Part1

This memorandum describes the process used to model shock and rarefaction waves propagating in a duct. First, a brief problem description will be given, followed by an overview of the governing equations. The results of the output will then be discussed and finally a copy of the computer code will be given.

A duct, 10 m in length with a cross-sectional area of .01 m^2, is filled with air, modeled as a calorically perfect ideal gas. At time t=0 s, the air is at rest with T= 300 K. A thin diaphragm, located in the center of the duct, separates two different pressure regions. On one side the pressure is 2000 kPa while the other side is at 100 kPa. At t=0+, this diaphragm breaks and, because of this drastic pressure difference, shock and rarefaction waves form and propagate through the duct. The objective of this code will be to model this process up until the time that the waves reach the ends of the duct. At this point the waves reflect, a phenomenon not considered in this code.

The equations which were solved to model this process are the conservation of mass, momentum and energy. In order to solve for the correct wave speed, it was necessary to cast these equations in their conservative form; the resulting equations are presented here. The conservative form of the mass equation is

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 \ , \tag{0.1}$$

where ρ is the density, u is the velocity, x is the position variable, and t is the time variable. In the same manner, the conservative for of the momentum equation can be formed:

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + P) = 0 \ , \tag{0.2}$$

where P represents the pressure. Finally, the conservative form of the energy equation is

$$\frac{\partial}{\partial t}\left(\rho\left(\frac{1}{\gamma - 1}\frac{P}{\rho} + \frac{u^2}{2}\right)\right) + \frac{\partial}{\partial x}\left(\rho u\left(\frac{1}{\gamma - 1}\frac{P}{\rho} + \frac{u^2}{2} + \frac{P}{\rho}\right)\right) = 0 \ , \tag{0.3}$$

where γ is a constant. These equations were solved using the Lax-Wendhoff numerical solution technique. This method involves iterating these equations by first finding values at half time steps and then using these values to predict values at the full time step. This code is provided in Appendix A. The results obtained from this numerical solution are presented in the following paragraph.

Figure 0.1 shows the pressure distribution at .4 ms. By this time, a shock wave can be seen propagating to the right as a rarefaction, or expansion, wave moves to the left. As the name suggests, the expansion wave converts high pressure air to a lower pressure. Also, pressure is seen to increases behind the shock wave. Figure 0.2 is a plot of the velocity distribution at .4 ms. At this time, the entire duct has not yet been excited. Only the air located inside the expansion wave-shock wave boundary is moving. Finally, Figure 0.3 shows the density distribution in the duct. This plot also supports the conclusion that the wave propagating to the left is an expansion wave and the wave moving to the right is a shock wave; the density decreases as it moves through the rarefaction and increases discontinuously as it moves through the shock.

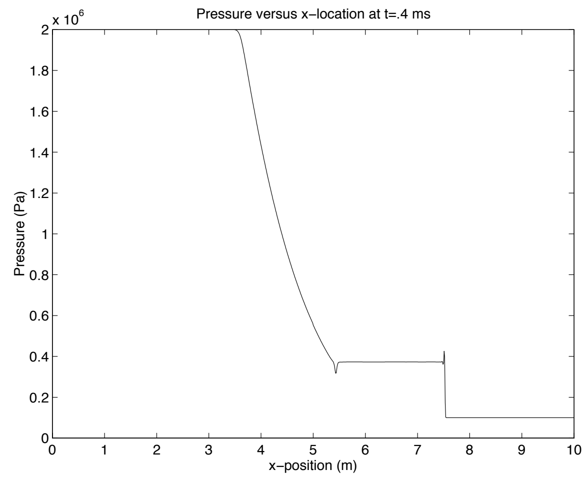

Figure 0.1: Pressure distribution in the shock tube at .4 ms.

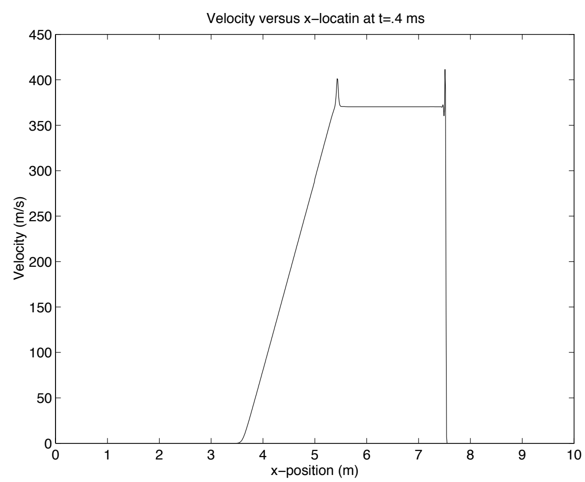

Figure 0.2: Velocity distribution in the shock tube at .4 ms.

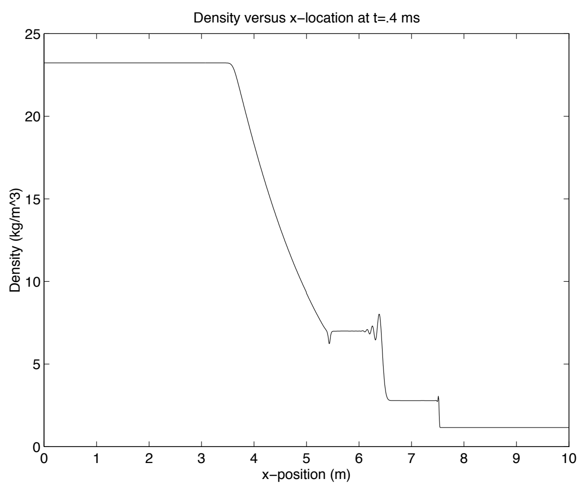

Figure 0.3: Density distribution in the shock tube at .4 ms.