ME332
FLUID MECHANICS LABORATORY
(PART II)

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Contents

Unit 5: Momentum transfer

(a) Thrust of propeller

The objective is to measure the static thrust developed by a ducted propeller at different rotational rates and compare to theory. The *actuator disk theory* of Rankine (Section 10-5.1b of *Introduction to Fluid Mechanics,* R.W. Fox and A.T. McDonald, 5th Ed., John Wiley, 1998) for a free, unducted propeller is *not* applicable here. A schematic of the experiment is shown in Fig. 1.

For steady flow, the force on the fluid in a control volume CV is given by equation (4.18)

$$\vec{F} = \int_{CS} \vec{V} \rho \vec{V} \cdot d\vec{A} \tag{1}$$

where CS is the surface of CV. The suggested control volume is indicated by dashed lines in the figure. The component of the equation in the flow direction is

$$F = \int_{CV} \rho V^2 \, dA \tag{2}$$

Assuming a uniform velocity[1] across sections 1 and 2, this reduces to

$$F = \left(-\rho_1 \overline{V}_1^2 + \rho_2 \overline{V}_2^2\right) A \tag{3}$$

Since $\rho_1 V_1 A = \rho_2 V_2 A = \dot{m}$ by mass conservation, we get

$$F = \dot{m}(-\overline{V}_1 + \overline{V}_2) \tag{4}$$

Because the density change of the air is small, we can neglect the change in fluid velocity. Thus, the sum of forces on the control volume due to the propeller and the pressures must be zero. From this we get

$$T = (p_2 - p_1)\ A \tag{5}$$

where T is the thrust, p_i is the pressure at i, and A is the cross-sectional area. The force of the propeller on the fluid is towards the right in the figure, and that of the fluid on the propeller towards the left. The pressure difference between inlet and outlet is indicated by the differential pressure transducer; the force on the propeller is measured by a load cell. Compare the measured values of the thrust with the theoretical values.

A blade-passing signal is provided by a LED-photocell pair[2]; its frequency can be measured from which the rotational rate of the propeller can be determined. Notice that the blades cut the light beam twice in a revolution. Plot the propeller thrust as a function of its rotational rate. Equation (10.33) of *Introduction to Fluid Mechanics,* R.W. Fox and A.T. McDonald, 5th Ed., John Wiley, 1998 predicts that, on dimensional grounds, the thrust should be proportional to the square of the rpm.

[1] If the velocity is not uniform, as may be the case especially in the downstream location, local velocities have to be measured and the integral obtained numerically. Here we will assume the velocity to be uniform.

[2] LED = Light Emitting Diode.

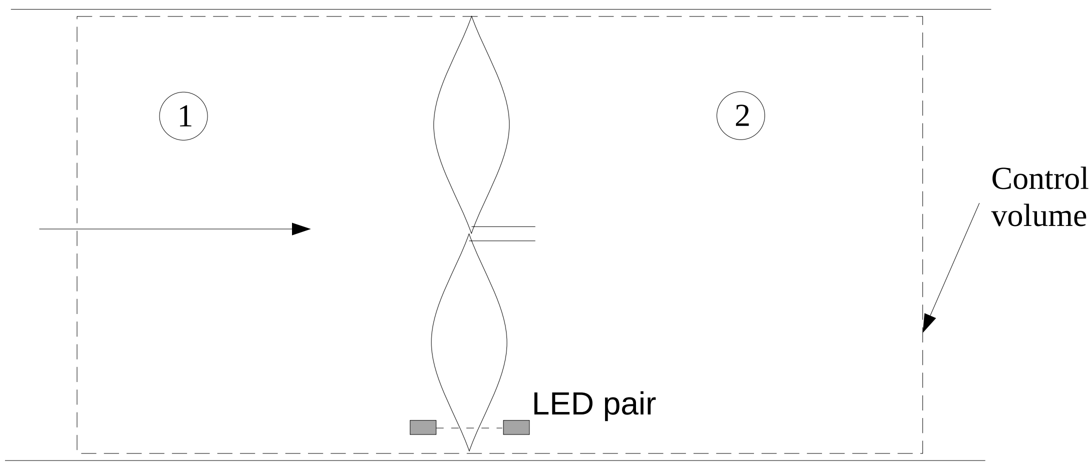

Figure 1: Schematic of propeller thrust experiment.

(b) Force of jet

The objective is to measure the force of a water jet on a flat surface and to compare the results to theory. The jet is schematically shown in Fig. 2. Recommended reading: Section 4-4, and Example Problem 4.4 of *Introduction to Fluid Mechanics,* R.W. Fox and A.T. McDonald, 5th Ed., John Wiley, 1998.

Consider the control volume indicated in the diagram where we will apply the steady-state momentum equation (1). Momentum flux in the vertical direction exists only at the entrance. If the flow rate is high enough, the fluid leaves the plate horizontally so that at the outlet of the control volume there is no component of the flow velocity in the vertical direction. The pressure at the boundary of the control volume is everywhere atmospheric, so the force due to that is zero. There is a force that the plate exerts on the fluid in the control volume, T, and this is downwards. The gravity force on the fluid, W, is also downwards. Thus, taking the positive direction to be upwards, we have

$$-T - W = -\rho\overline{V}^2 A \tag{6}$$

where ρ and $\overline{V}$ are the density and the mean velocity of the fluid, respectively, and A is the cross-sectional area of the jet. W is negligible since 1 cm^3 of water weighs less than 0.01 N, and T is much larger than that (if not, then W has to be included). Another way of writing equation (6) is

$$T = \frac{\rho Q^2}{A} \tag{7}$$

where Q is the volume flow rate. As a reminder, this formula is applicable only when the flow out of the control volume is in the horizontal direction and the weight of the fluid is small. The volume flow rate is measured by a rotameter and the force on the flat plate by a load cell. The flow rate can be varied to obtain a force vs. flow rate curve. The theoretical and experimental values can be compared.

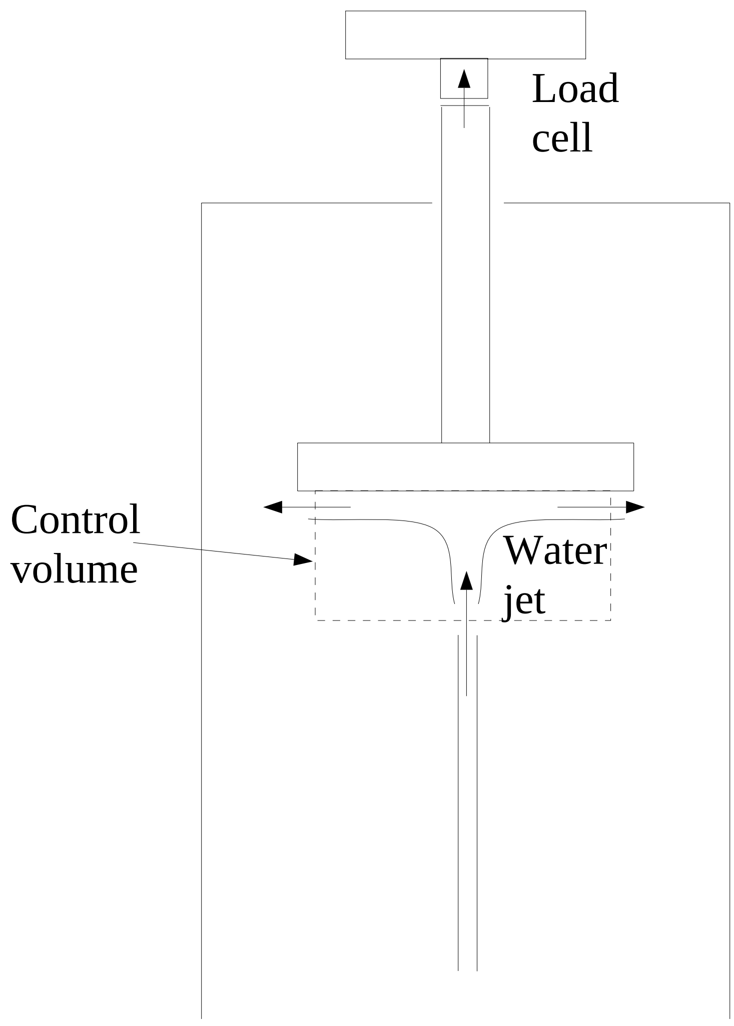

Figure 2: Schematic of force due to a water jet.

Unit 6: Viscometry

(a) Capillary viscometer

The objective is to determine the viscosity of water from the volume flow rate in a tube. The critical Reynolds number for transition can also be found. A schematic of the flow is shown in Fig. 3. The water is gravity driven from an overhead tank. The pressure drop and the volume flow rate can be measured. The manometer measures the gage pressure in the setting chamber. The volume flow rate is measured by timing a certain volume of liquid in a graduated cylinder. Recommended reading: Section 8-3, Fully developed laminar flow in a pipe.

Laminar flow

For laminar flow, the flow rate through a horizontal pipe of circular cross section is given by

$$Q = \frac{\pi D^4}{128\mu L}\,\Delta p \tag{8}$$

where Q is the volumetric flow rate, $\Delta p = p_1 - p_2$ is the pressure drop, L is the distance between the pressures, D is the pipe inner diameter, and μ is the viscosity of the fluid. Entrance effects have been neglected.

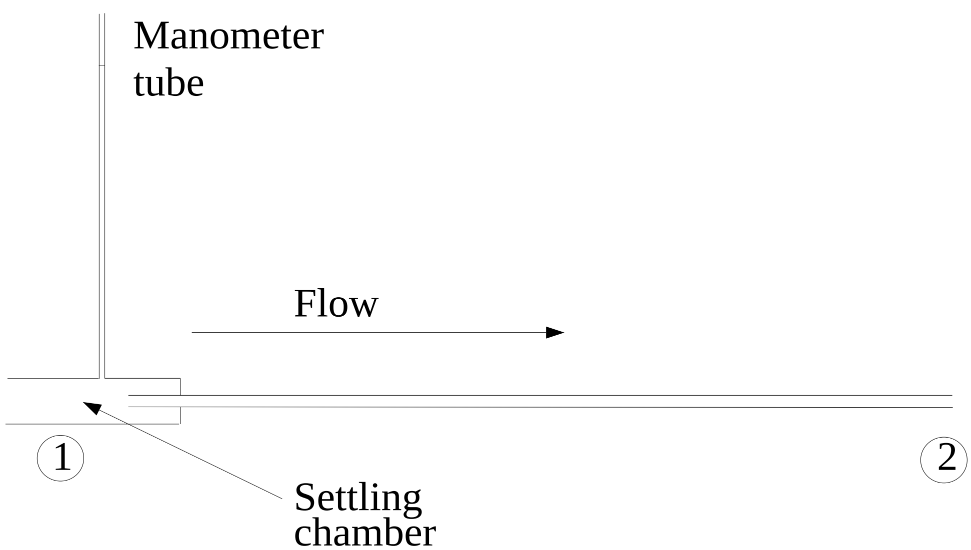

Figure 3: Flow in capillary tube

Transition to turbulence

The pressure drop versus flow rate relation for laminar flow in a pipe is linear, while for turbulent flow it is not. Thus the Δp-Q curve will indicate transition.

(b) Brookfield viscometer

In this instrument, also called Synchro-Lectric viscometer, the torque needed to rotate a cylinder or spindle is measured. Within a limited range of speeds, the torque, T, is proportional to the fluid viscosity, μ, and the rotation rate, ω, i.e.

$$T = K\mu\omega \tag{9}$$

Since the geometry of the container in which the fluid is kept is not simple, K cannot be determined analytically.

The cylinder is driven by a synchronous electric motor through an 8-speed gearbox and a spiral spring, schematically shown in Fig. 4. The spring is a metallic strip in the form of a spiral; the motor shaft is attached to the outer edge of the spiral and the cylinder shaft to its center. The rotation rate of the cylinder is precisely known for the 8 settings. The angular deflection of the spring, measured by the pointer against the graduated disk, is proportional to the torque being transmitted. Additional information about this viscometer can be found in: http://www.brookfieldeng.com/support/documentation/index.cfm. It is a Brookfield dial viscometer, also known as an LVT viscometer.

The instrument calibration constant K is found by using a fluid of known viscosity, after which the viscosity of any other fluid can be experimentally determined. The objective of this experiment is to measure the viscosity of a given fluid at different temperatures. The fluid is placed on an electric heater and its viscosity and temperature are measured at the same time while it is being heated. A thermometer immersed in the fluid gives its temperature.

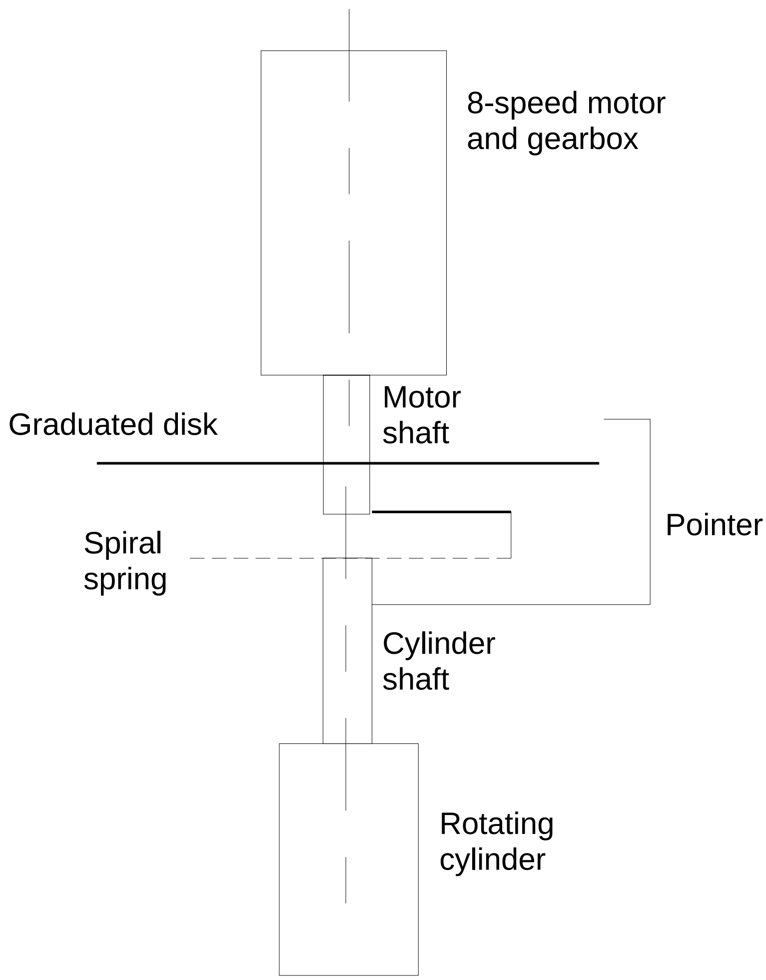

Figure 4: Brookfield viscometer.

Unit 7: Minor losses

(a) Valves

The objective is to determine the loss coefficients for several valves as a function of valve openings. The flow rate is measured by a rotameter, and a differential pressure transducer measures the pressure drop. The valve openings are measured in terms of angle, or better as fractions of the fully open angle. The valves available are: ball, gate, globe, needle and stop, schematic drawings of which are shown in Fig. 5. A layout of the valves is shown in Fig. 6. Recommended reading: Section 8-7.2.

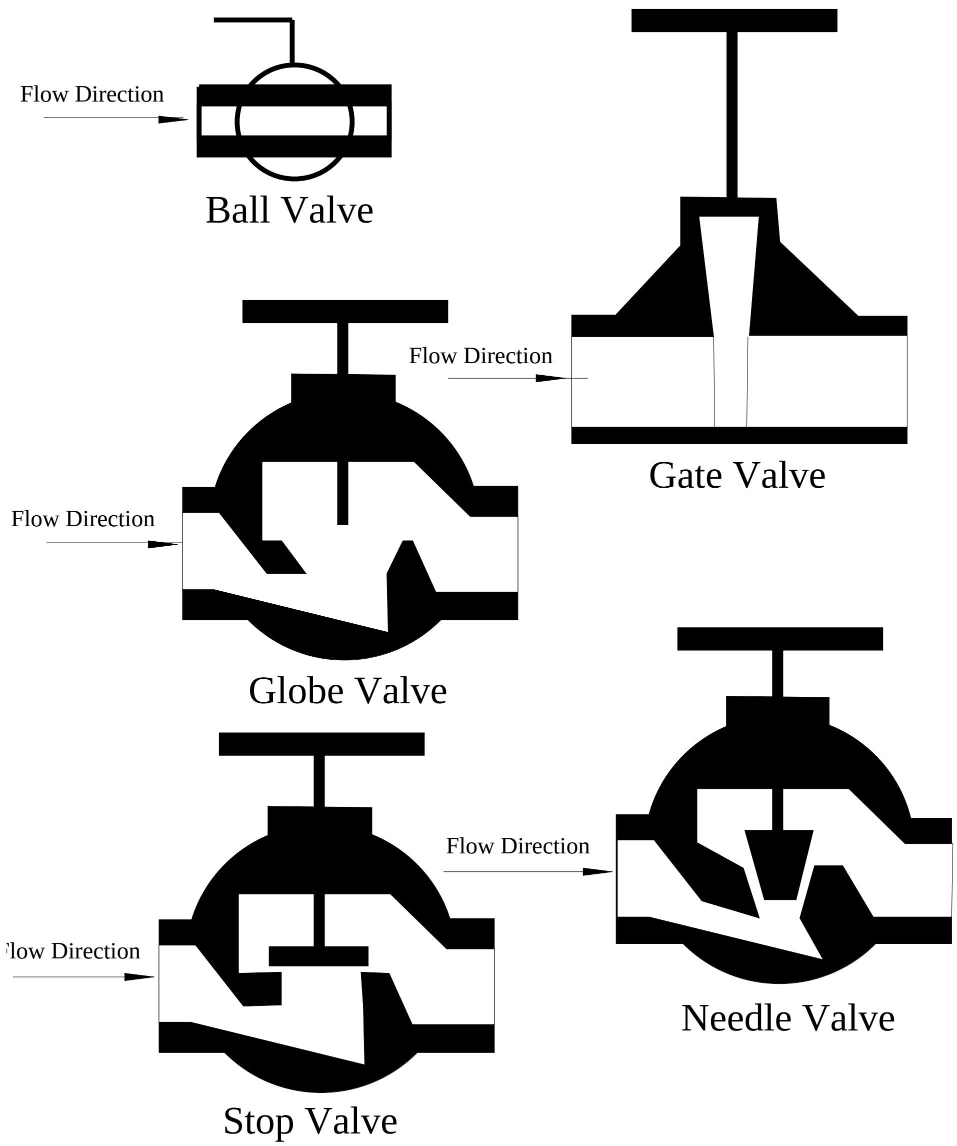

Figure 5: Sectional views of various valves.

The loss coefficient, K, is defined in equation (18).

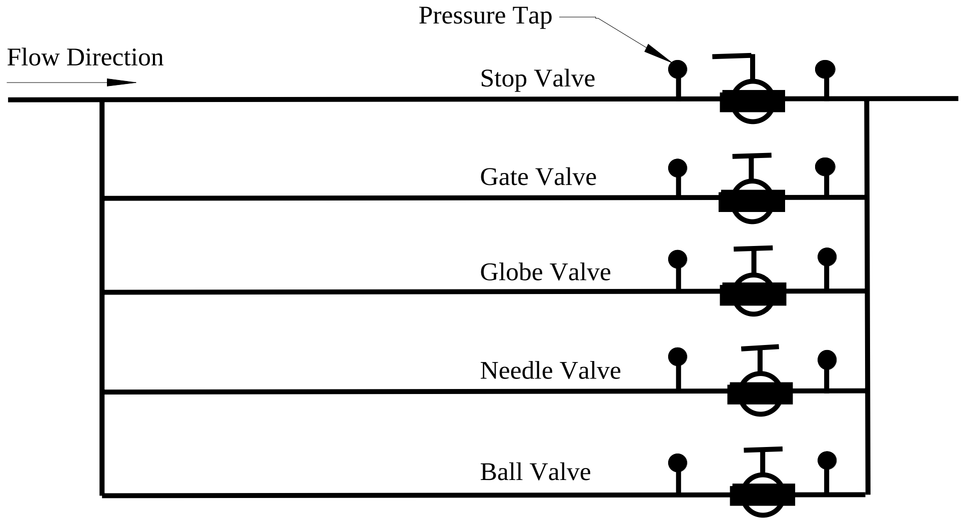

Figure 6: Location of valves.

(b) Fittings

The objective is to determine the loss coefficients and equivalent pipe lengths for several fittings. The fittings available are (i) a tee-junction (that can be tested in three different ways), (ii) a 90° elbow, (iii) a reduction and (iv) an expansion. A straight pipe is also available to determine the friction factor from pressure drop measurements using equation (20). The positions of the different fittings are indicated in Fig. 7. Recommended reading: Section 8-7.2.

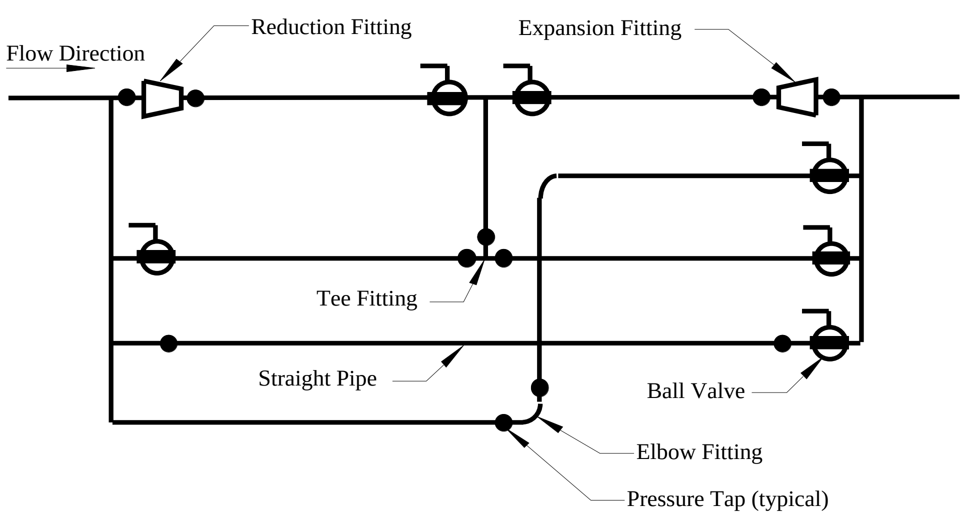

Figure 7: Location of fittings.

Equation (18) is valid for the tee-junction and the elbow for which the mean velocity does not change. For the reduction and expansion, however, there are area changes and consequent changes in mean velocity. In this case the loss coefficient, K, may be defined in terms of the mean velocity, $\overline{V}_s$, at the smaller of the two sections.

Combining this with equation (17) where $z_1 = z_2$, we get

$$\frac{p_1 - p_2}{\rho g} + \frac{\overline{V}_1^2 - \overline{V}_2^2}{2g} = K \ \frac{\overline{V}_s^2}{2g} \tag{10}$$

instead of equation (18), where 1 is upstream and 2 downstream of the fitting. The loss coefficient can be calculated from this equation. For an expansion $\overline{V}_s = \overline{V}_1$ and for a contraction $\overline{V}_s = \overline{V}_2$.

Unit 8: Energy transfer

(a) Centrifugal pump

The objective is to determine the characteristic curves for the head and the theoretical power versus flow rate curve for different pump speeds. The efficiency is not measured here. Determine also the nondimensional and dimensional specific speeds. Recommended reading: p. 318, p. 527, Fig. 10.29, Section 10-4, Appendix D of *Introduction to Fluid Mechanics,* R.W. Fox and A.T. McDonald, 5th Ed., John Wiley, 1998; p. 163, *Fundamentals of Engineering Thermodynamics*, M. J. Moran and H. N. Shapiro, 4th Ed., John Wiley, 2000.

The pump is mounted in a loop shown in Fig. 8. There is an air vent to remove any air in the loop. The flow rate is varied by a butterfly valve; the flow rate is measured by a paddle-wheel flow meter the output of which is read by a panel meter. The difference in pressures between the inlet and outlet of the pump is measured by a differential transducer and measured by another panel meter. The pump speed can be varied and the frequency of the voltage is proportional to its rpm. The water for the loop comes from the building water line and there is a double-check reduced pressure vent system to prevent water from going back into the line (this is always necessary when installing a pump) and an expansion tank for changes in volume due to temperatures.

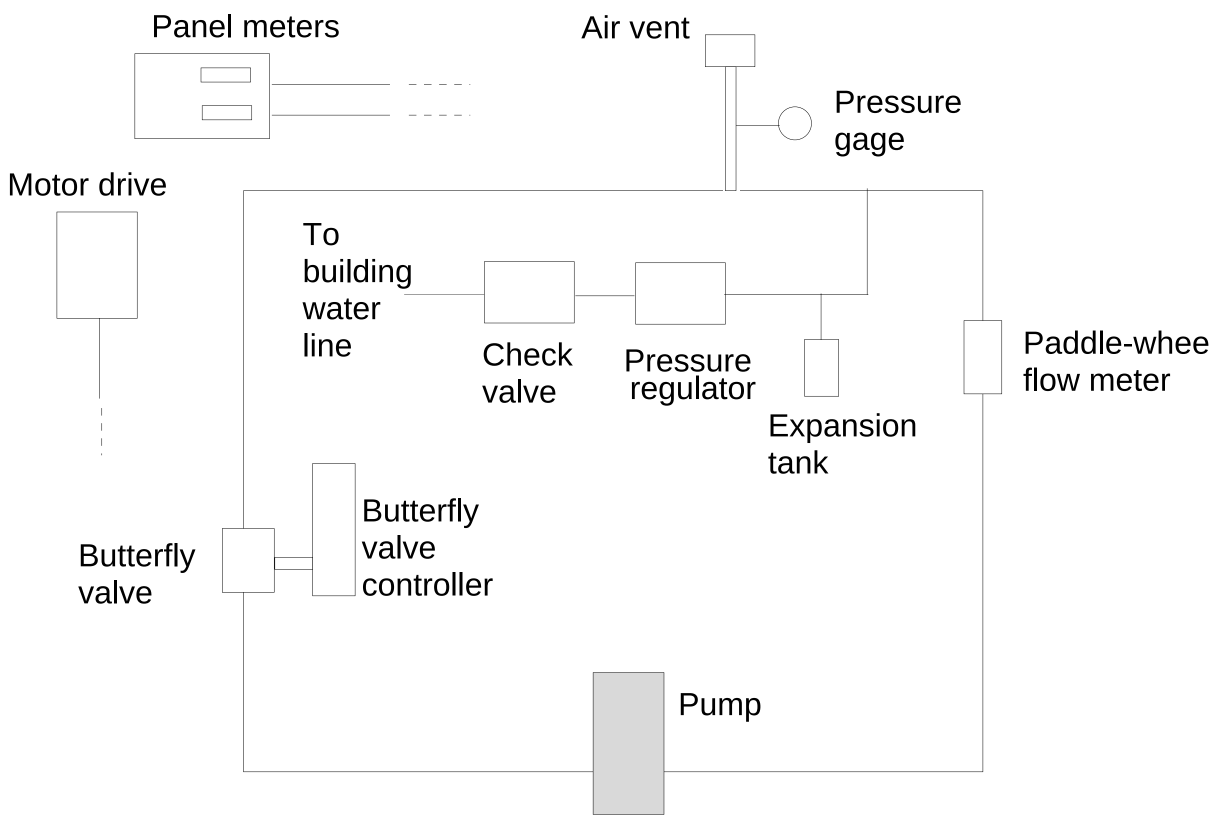

Figure 8: Schematic of test loop.

The pump is schematically shown in Fig. 9. For an incompressible fluid such as

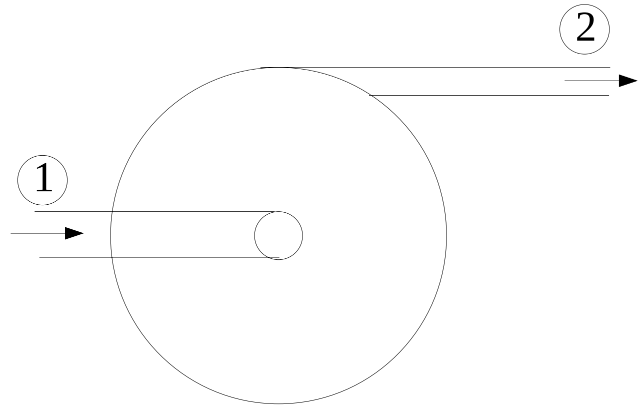

Figure 9: Schematic of centrifugal pump.

water, the change in enthalpy due to change in its internal energy may be negligible. Since the enthalpy is defined as

$$h = u + \frac{p}{\rho} \tag{11}$$

where u is the internal energy, p is the pressure, and ρ is the fluid density, the expression for the head, equation (21), simplifies to

$$H = \left(\frac{p}{\rho g} + \frac{\overline{V}^2}{2g} + z \right)_2 - \left(\frac{p}{\rho g} + \frac{\overline{V}^2}{2g} + z \right)_1 \tag{12}$$

(b) Heat exchanger

The objective is to check for energy balance in a concentric tube heat exchanger in parallel and counterflow. Hot and cold water from the tap are used for the two sides of the heat exchanger. By closing and opening some valves, either parallel flow or counterflow can be obtained. The temperatures are measured using type T thermocouples (i.e. copper-constantan) with an ice-point cell as reference. Remember that it takes some time for all parts of the apparatus to come to thermal equilibrium; so give it time to settle down. Recommended reading: Section 4.3.1, pp. 166–171, *Fundamentals of Engineering Thermodynamics,* M. J. Moran and H. N. Shapiro, 4th Ed., John Wiley, 2000.

(i) Parallel flow: In a parallel flow arrangement the two streams are in the same direction as shown in Fig. 10. The flow of hot water in the inner tube is designated by the subscript 1 and the cold water in the outer by 2. The heat and mass flow rates are $\dot{q}$ and $\dot{m}$; c is the specific heat at constant pressure of the fluid. The heat given up by the hot water is

$$\dot{q}_1 = \dot{m}_1 c (T^1_{in} - T^1_{out}) \tag{13}$$

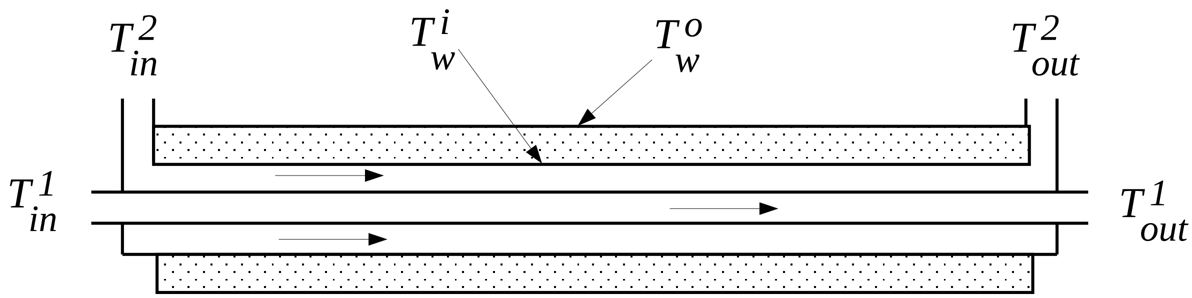

Figure 10: Schematic of heat exchanger in parallel flow.

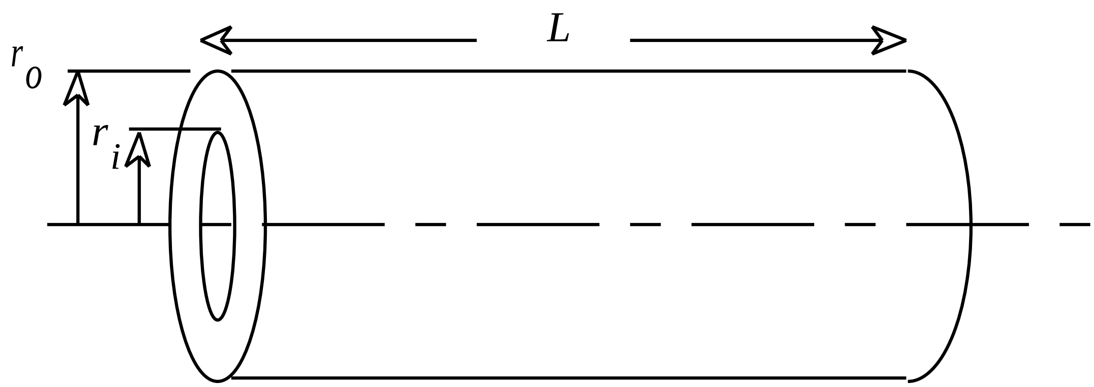

Figure 11: Geometry of the insulation.

since the velocity at the inlet and outlet do not change appreciably. The heat received by the cold water is similarly

$$\dot{q}_2 = \dot{m}_2 c(T^2_{out} - T^2_{in}) \tag{14}$$

Some heat is lost to the room. We can estimate the conduction through the cylindrical insulation as

$$\dot{q}_{cond} = \frac{2\pi k L(T^w_i - T^w_o)}{\ln(r_o/r_i)} \tag{15}$$

where k is the thermal conductivity of the insulation which must be given to you, and T^w_i and T^w_o are the inner and outer wall temperatures. Remember that $\dot{q}_{cond}$ will be negative if $T^w_o > T^w_i$. The geometry of the insulation is shown in Fig. 11, the dimensions of which can be measured.

For perfect heat balance we must have

$$\dot{q}_1 = \dot{q}_2 + \dot{q}_{cond} \tag{16}$$

The left and right sides of the equation can be determined and compared.

(ii) Counter flow: The counterflow arrangement is similar to the above, except that one of the flow directions is reversed.

Appendix

Load cell

The load cells used in this laboratory are Model LCL made by Omega. More information can be found in their Webpage:

http://www.omega.com/toc_asp/subsection.asp?subsection=F07&book=Pressure

Head change

The general definition of change in head between two sections 1 and 2 in a duct with different cross-sectional area is[3]

$$h_f = \left(\frac{p}{\rho g} + \frac{\overline{V}^2}{2g} + z \right)_1 - \left(\frac{p}{\rho g} + \frac{\overline{V}^2}{2g} + z \right)_2 \tag{17}$$

where 1 is upstream of 2. Here h_f is the head change, p is the pressure, ρ is the fluid density, $\overline{V}$ is the mean velocity, z is the height above a reference, and g is the acceleration due to gravity.

Minor losses

The loss coefficient, K, in a fitting is defined by

$$h_f = K \frac{\overline{V}^2}{2g} \tag{18}$$

where K is the loss coefficient and $\overline{V}$ is the mean velocity. There is no confusion if $\overline{V}$ is the same on either side of the fitting. If, however, it is different, as in a contraction, it should be specified which side $\overline{V}$ corresponds to.

The length of straight pipe that would have the same pressure drop is given by

$$L_{eq} = \frac{K}{f} D \tag{19}$$

where L_{eq} is the equivalent length, f is the friction factor and D is the diameter of the straight pipe.

Major losses

The friction factor f in fully developed flow of an incompressible fluid in a horizontal pipe of constant cross-sectional area is defined by

$$h_f = f \, \frac{L}{D} \, \left(\frac{\overline{V}^2}{2g} \right) \tag{20}$$

where $p_1 - p_2$ is the pressure drop along a length L of the pipe, and D is the inner diameter of the pipe. Section 1 is upstream of 2.

[3]Some authors multiply this expression by g. It is then no longer in units of length.

Pump characteristics

A working machine (such as a pump, compressor or fan) puts energy into a fluid stream while a power machine (like a turbine) takes energy out. The head of a fluid machine is defined as the rate of work put in or taken out per unit weight of fluid. It has units of length[4]. For a working machine it is

$$H = \left(\frac{h}{g} + \frac{\overline{V}^2}{2g} + z \right)_{outlet} - \left(\frac{h}{g} + \frac{\overline{V}^2}{2g} + z \right)_{inlet} \tag{21}$$

where H is the head, h is the specific enthalpy, g is the acceleration due to gravity, $\overline{V}$ is the mean velocity, and z is the height above a reference. We have ignored the heat transfer from the machine. Since $\rho g Q$ is the weight flow rate through the machine, the theoretical power to or from the machine is given by

$$P = \rho g Q H \tag{22}$$

where P is the power, ρ is the fluid density, and Q is the volume flow rate.

The fan or pump characteristic (see p. 317) is a diagram which has the head, power and efficiency curves versus the volume flow rate. The curves are usually drawn for a series of constant rotational rates of the machine.

The nondimensional specific speed for a working machine is given by

$$N_s^{nd} = \frac{\omega Q^{1/2}}{(gH)^{3/4}} \tag{23}$$

where ω is the rotation rate in rad/s, Q the volume flow rate in m^3/s, and H the head in m. There is also a commonly used dimensional specific speed

$$N_s^d = \frac{n Q^{1/2}}{H^{3/4}} \tag{24}$$

where now n is the rotation rate is in rpm, Q is the volume flow rate in gpm (gallons per minute), and H is the head in $feet$.

Type T thermocouple

The calibration of a type T thermocouple is

$$\begin{aligned} T \quad = \quad & \left(25.661297 \, \frac{C}{mV}\right) E - \left(0.61954869 \, \frac{C}{mV^2}\right) E^2 \\ & + \left(0.022181644 \, \frac{C}{mV^3}\right) E^3 - \left(0.00035500900 \, \frac{C}{mV^4}\right) E^4 \end{aligned} \tag{25}$$

where T is the temperature in Celsius, and E is the thermocouple emf in millivolts.

[4]Remember that some books use a slightly different definition in which the right side is multiplied by g so that the head comes out in different units.