AME 332: Fluid Mechanics Laboratory
Solution
Midterm Examination
Prof. J. M. Powers
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1. (5) Write an appropriate first sentence for the abstract of Unit 2, Flow Visualization.

Solution

There are many varieties of first sentences. Qualities which I sought were efficiency, good grammar, good sentence structure, completeness, lack of ambiguity, and direct application to the laboratory. A good first sentence might be

"Laminar, turbulent, and flows in transition were studied with dye injection visualization techniques for internal and external flows."

Common errors included use of multiple sentences, lack of a comprehensive statement, fuzzy sentence structure, and use of statements which were too broad and imprecise.

2. (20) Write a short paragraph which identifies Bernoulli's equation and explains its significance. Your explanation will be assessed for both its technical and linguistic merits, consistent with the expectations of AME 332.

Solution

Qualities which were viewed in a positive light here included a correct identification of Bernoulli's equation, an identification of the variables and constants in the equation, a discussion of the physical significance and physical origins of the equation, a discussion of the assumptions necessary for its use, and a possible application of the equation.

A good paragraph might read as follows:

"Bernoulli's equation has fundamental importance in fluid mechanics. There is not universal agreement on the best way to express it, but one common form is

$$\frac{p}{\rho} + \frac{\mathbf{v} \cdot \mathbf{v}}{2} + gz = C. \tag{1}$$

Here p is the pressure, ρ is the density, $\mathbf{v}$ is the velocity vector, z is the height, g is the constant gravitational acceleration, and C is a constant (valid for all space and time). The equation is a statement of the conservation of linear momentum of the fluid for the special case in which the flow is steady, incompressible, inviscid, and irrotational. Slight relaxation of these assumptions can lead to other versions of Bernoulli's equation. It can also be interpreted as a statement of conservation of mechanical energy of the flow where the term $\mathbf{v} \cdot \mathbf{v}/2$ represents the specific kinetic energy, the term gz represents the gravitational potential energy, and p/ρ is a potential energy associated with the fluid's pressure. A variety of physical phenomena can be understood by simple application of the Bernoulli equation and its variants including flow over airfoils and flow in ducts."

Common deficiencies included imprecise identification of the equation, using words which did little more than repeat precisely what the equation itself said, using wordy statements which said little more than knowing all variables except one, the other can be determined, and lack of any discussion of the origins, assumptions, or physical interpretation of the equation.

3. (20) For turbulent flow in pipes at sufficiently low Reynolds number, Re_D, the Blasius correlation for the friction factor f is appropriate:

$$f = \frac{0.316}{Re_D^{0.25}}, \qquad Re_D < 10^5.$$

If $v = 10 \pm 0.3\ m/s$, $D = 0.01 \pm 0.005\ m$, $\nu = 1 \times 10^{-5} \pm 1 \times 10^{-6}\ m^2/s$, find f and, using the method developed in class involving partial derivatives, estimate the error in f.

Solution

First find f.

$$\begin{aligned} Re_D &= \frac{vD}{\nu} = \frac{(10)(0.01)}{10^{-5}} = 10^4, \\ f &= 0.316 Re_D^{-1/4} = 0.316\left(10^4\right)^{-1/4} = 0.0316. \end{aligned}$$

Now get the uncertainty.

$$\begin{aligned} f &= 0.316\left(\frac{\nu}{vD}\right)^{1/4}, \\ f &= 0.316\nu^{1/4}v^{-1/4}D^{-1/4}, \\ \Delta f &= \frac{\partial f}{\partial \nu}\Delta\nu + \frac{\partial f}{\partial v}\Delta v + \frac{\partial f}{\partial D}\Delta D, \\ \Delta f &= 0.316\left(\frac{1}{4}\nu^{-3/4}v^{-1/4}D^{-1/4}\Delta\nu - \frac{1}{4}\nu^{1/4}v^{-5/4}D^{-1/4}\Delta v - \frac{1}{4}\nu^{1/4}v^{-1/4}D^{-5/4}\Delta D\right), \\ \Delta f &= \frac{0.316}{4}\left(\frac{\nu}{vD}\right)^{1/4}\left(\frac{\Delta\nu}{\nu} - \frac{\Delta v}{v} - \frac{\Delta D}{D}\right). \end{aligned}$$

Assuming the errors are uncorrelated, we then estimate the error as

$$\begin{aligned} \Delta f &= \frac{0.316}{4}\left(\frac{\nu}{vD}\right)^{1/4}\sqrt{\left(\frac{\Delta\nu}{\nu}\right)^2 + \left(\frac{\Delta v}{v}\right)^2 + \left(\frac{\Delta D}{D}\right)^2}, \\ \Delta f &= \frac{0.316}{4}\left(10^{-4}\right)^{1/4}\sqrt{\left(\frac{10^{-6}}{10^{-5}}\right)^2 + \left(\frac{0.3}{10}\right)^2 + \left(\frac{0.005}{0.01}\right)^2}, \\ \Delta f &= 0.004035 \end{aligned}$$

So

$$f = 0.0316 \pm 0.004035.$$

The most common error here was a lack of fundamental understanding of how to compute the error using the partial derivative expansion. There were also several calculational errors.

4. (20) Water, $\rho = 997\ kg/m^3$, flows in a slot of height $0.003\ m$. The width of the slot is sufficiently large so that three dimensional effects can be neglected, and the flow can be taken to be steady and fully developed in the streamwise direction. The velocity parallel to the slot wall at a point $0.001\ m$ above the bottom of the slot is measured to be $0.11\ m/s$, and the velocity at a point $0.002\ m$ above the bottom of the slot is measured to be $0.08\ m/s$. Estimate the mass flow rate per unit width of the slot.

Solution

Assuming the flow is steady, the mass flow rate is obtained by the following integral

$$\dot{m} = \int_0^h \rho \mathbf{v} \cdot \mathbf{n} dA.$$

Here $\dot{m}$ is the mass flow rate, ρ is the fluid density, $\mathbf{v}$ is the fluid velocity vector, $\mathbf{n}$ is the unit normal vector to the cross sectional area of interest, h is the height of the slot, and dA is the differential area. Defining the depth of the slot to be b, taking the cross-sectional area to be normal to the slot, as well as the velocity, and taking y to be the distance variable which traverses the slot, taking the velocity vector to be $\mathbf{v} = u(y)$, and taking the density to be constant, we reduce the formula to

$$\dot{m} = \rho b \int_0^h u(y)\ dy.$$

There are a variety of ways to evaluate the integral $\int_0^h u(y)\ dy$. Importantly, one should account for the fact the wall boundaries have zero velocity. A straightforward way to evaluate the integral is the trapezoidal rule:

$$\int_0^h u(y)\ dy \sim \frac{\Delta y}{2} \left(u_0 + 2u_1 + 2u_2 + u_3 \right).$$

$$\int_0^h u(y)\ dy \sim \frac{0.001}{2} \left(0 + 2(0.08) + 2(0.11) + 0 \right) = 0.00019\ \frac{m^2}{s}.$$

So, we get

$$\frac{\dot{m}}{b} = \rho \int_0^h u(y)\ dy = \left(997\ \frac{kg}{m^3} \right) \left(0.00019\ \frac{m^2}{s} \right) = 0.189\ \frac{kg}{m\ s}.$$

There are many other ways to evaluate the integral, none of which require memorization of the trapezoidal rule. Simply modeling the area under the curve as a set of rectangles, and adding the areas of the rectangles gives a good approximation.

5. (10) The equation for the free surface of a rotating fluid, $z = \frac{\omega^2 r^2}{2g}$, is valid for turbulent flow, after appropriate averages are taken into account. TRUE or FALSE; give a short defense of your answer.

Solution

FALSE. The equation is derived from a limiting case of the Navier-Stokes equations in which the fluid is rotating as a rigid body. Hence there can be no convection, no viscous stress, no turbulence, and no unsteadiness. The only mechanisms in play from the linear momentum equation are pressure gradients, body forces, and centripetal acceleration.

6. (10) As viscosity increases, all else held equal, the Strouhal number will remain constant. TRUE or FALSE; give a short defense of your answer.

Solution

FALSE. While for $Re > 1000$ it is an empirical fact that St is roughly constant, it is also an empirical fact that as $Re \rightarrow 0$, $St \rightarrow 0$. Physically this means that very viscous flows do not shed vortices. One can imagine the flow of honey over a salt shaker and easily accept the fact that it would be unlikely to see a vortex shed for typical flow conditions which could be achieved.

7. (10) The equation to determine velocity in a total head probe

$$v = \sqrt{\frac{2(p_o - p)}{\rho}},$$

is valid for compressible flows. TRUE or FALSE; give a short defense of your answer.

Solution

FALSE. This equation was derived from a limiting case of Bernoulli's equation in which the density was constant. A different version of this equation could be developed for compressible flows.

8. (10) Fluid particles in the entrance region of pipe flow are accelerating. TRUE or FALSE; give a short defense of your answer.

Solution

TRUE. By definition, a fluid particle in the entrance region of a pipe has not yet entered the fully developed region in which there is no variation in the axial direction. So in general, a fluid particle in the entrance region will experience a change in its velocity as it travels through the region, so it is accelerating.

The following question has nothing to do with this course and is included solely to see how ND engineers do compared to 168 physics and engineering students at Iowa State. Your answers on this will not influence your grade in any way. On this question 120 out of 168 Iowa State students got the correct answer. I'm thinking we can do better! The question is

If a pen is dropped on the moon, will it
a) float away,
b) float where it is,
c) fall to the surface of the moon.

Solution

Well, it is C. I think there were five incorrect answers, although one of the incorrect answers was given in a spirit of contrarian protest. So I think we topped the Cyclones on this.