Deriving Features of Reacting Hypersonic Flow from Gradients at a Curved Shock

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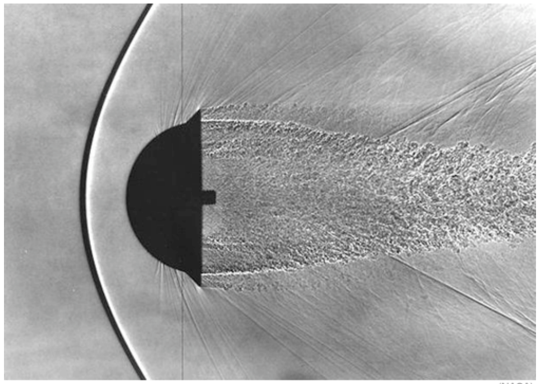


(NASA)

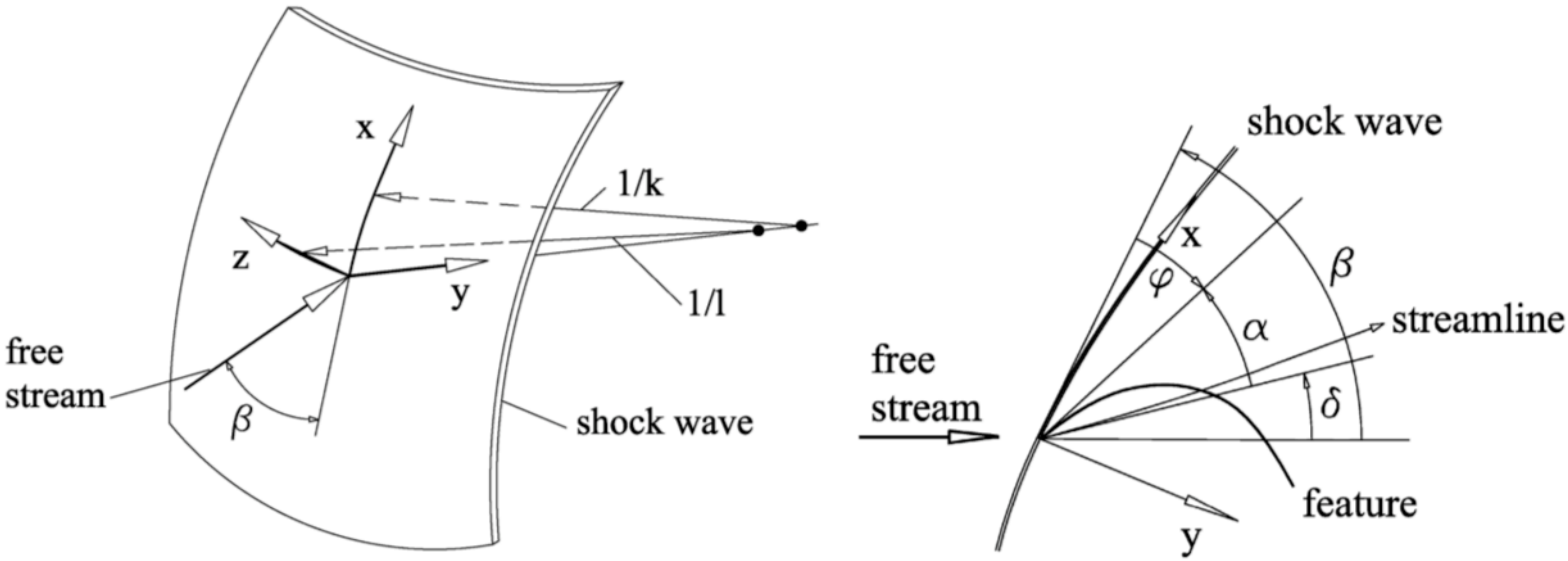
Outline

- Purpose of Derivation
- Coordinate System
- Gas Properties and Equations of Motion
- Shock Jump Relations
- Simplified Differential Equations
- Reaction Model
- Property Fields as Related to Reaction Rate

Purpose of Derivation

- Shock waves cause flow variables to change discontinuously.
- Curvature/Acceleration of a shockwave, non-uniform free stream conditions, and reaction rates cause downstream variables to be non-uniform.
- **Purpose**: Use the behaviors and gradients of flow variables at a shock to determine the field of a flow variable in a region downstream of the shock.

Coordinate System



β = Shock Angle

δ = Deflection Angle

α = Angle between flow direction (streamline) and feature direction (e.g. entropy contour)

Thermodynamic Properties

Assume *h* and *T* obey caloric and thermal equations of state:

$$h = h(p,\rho,c_i)$$

$$T = T(p,\rho,c_i)$$

Where *p* is the pressure, *ρ* is the density, and c_i is the mass fraction where

$$\sum_{i=1}^{n} c_i = 1$$

This allows us to derive the change in enthalpy:

$$dh = \frac{\partial h}{\partial \rho}\Big|_{p,c_i} d\rho + \frac{\partial h}{\partial p}\Big|_{\rho,c_i} dp + \sum_{i=2}^{N} \frac{\partial h}{\partial c_i}\Big|_{p,c_j,j\neq i} dc_i = h_\rho d\rho + h_p dp + \sum_{i=2}^{N} h_{c_i} dc_i$$

Equations of Motion

Assumptions:

1. Steady Flow
2. Uniform Flow in Free Stream
3. All *w* and *z* derivatives are zero (except w_z)
4. Interested in area <u>at</u> shock wave (y = 0)

$$uu_x + vu_y - kuv + p_x / \rho = 0$$

$$uv_x + vv_y + ku^2 + p_y / \rho = 0$$

} Form of Navier Stokes

$$h_p + h_\rho \rho_y + \sum_{i=2}^{n} h_{c_i} c_{iy} + vv_y + uu_y = 0$$

$$h_p p_x + h_\rho \rho_x + uu_x + vv_x = 0$$

} Conservation of Energy

$$(\rho u)_x - (k - l)\rho v + \rho l \sin\beta + (\rho v)_y = 0$$

Conservation of Momentum

Shock Jump Relations

$$p - p_\infty = \sin^2\beta(1 - 1/\rho)$$

$$c_i = c_{i\infty}$$

$$v = \sin\beta/\rho$$

$$2(h - h_\infty) = \sin^2\beta(1 - 1/\rho^2)$$

$$u = \cos\beta$$

$$\rho = \frac{\gamma + 1}{\gamma - 1 + 2/(M^2\sin^2\beta)}$$

- "Boundary" conditions for the equations of motion.
- Interesting note: The shock jump relations take place before any reaction occurs, therefore no changes in mass fractions.

γ = ratio of Specific Heats (γ = 1.4)

M = Mach Number (M = 6)

Simplified Differential Equations

$$p_y = A_{(p)}(M,\gamma,\beta)r + B_{(p)}(M,\gamma,\beta)l + C_{(p)}(M,\gamma,\beta)k$$

$$u_y = C_{(u)}(M,\gamma,\beta)k$$

$$v_y = A_{(v)}(M,\gamma,\beta)r + B_{(v)}(M,\gamma,\beta)l + C_{(v)}(M,\gamma,\beta)k$$

$$\rho_y = A_{(\rho)}(M,\gamma,\beta)r + B_{(\rho)}(M,\gamma,\beta)l + C_{(\rho)}(M,\gamma,\beta)k$$

$$r = v\sum_{i=2}^{N} h_{c_i} c_{iy} = \sum_{i=2}^{N} h_{c_i} \frac{dc_i}{dt}$$

l = Transverse Shock Curvature

k = Flow-Plane Shock Curvature

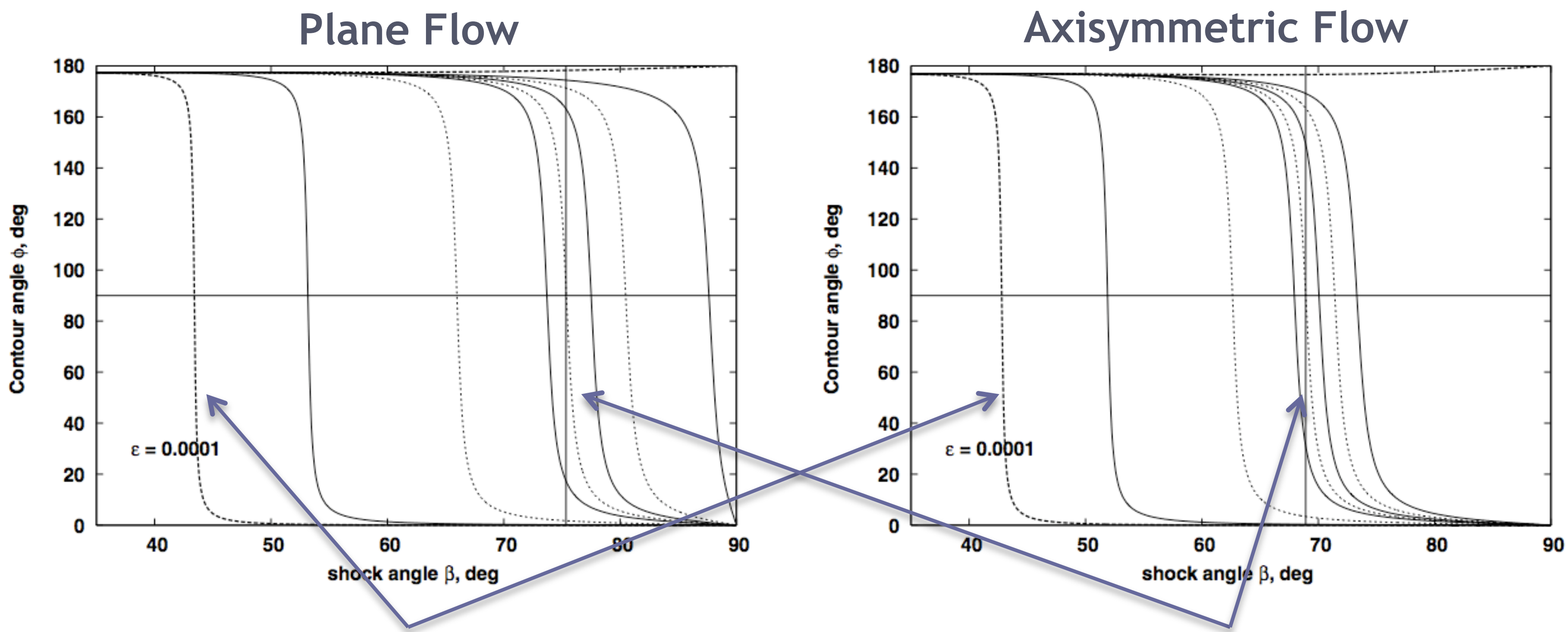
Reaction Model

- Based on dissociating molecules in a hypersonic flow
- Can be either endothermic or exothermic
- Model of Arrhenius Form:

$$r = \frac{\theta}{\varepsilon}\exp(-\theta\rho / p)$$

where θ is the dissociation energy (θ = 0.95), and ε is a rate constant (treated as a parameter).

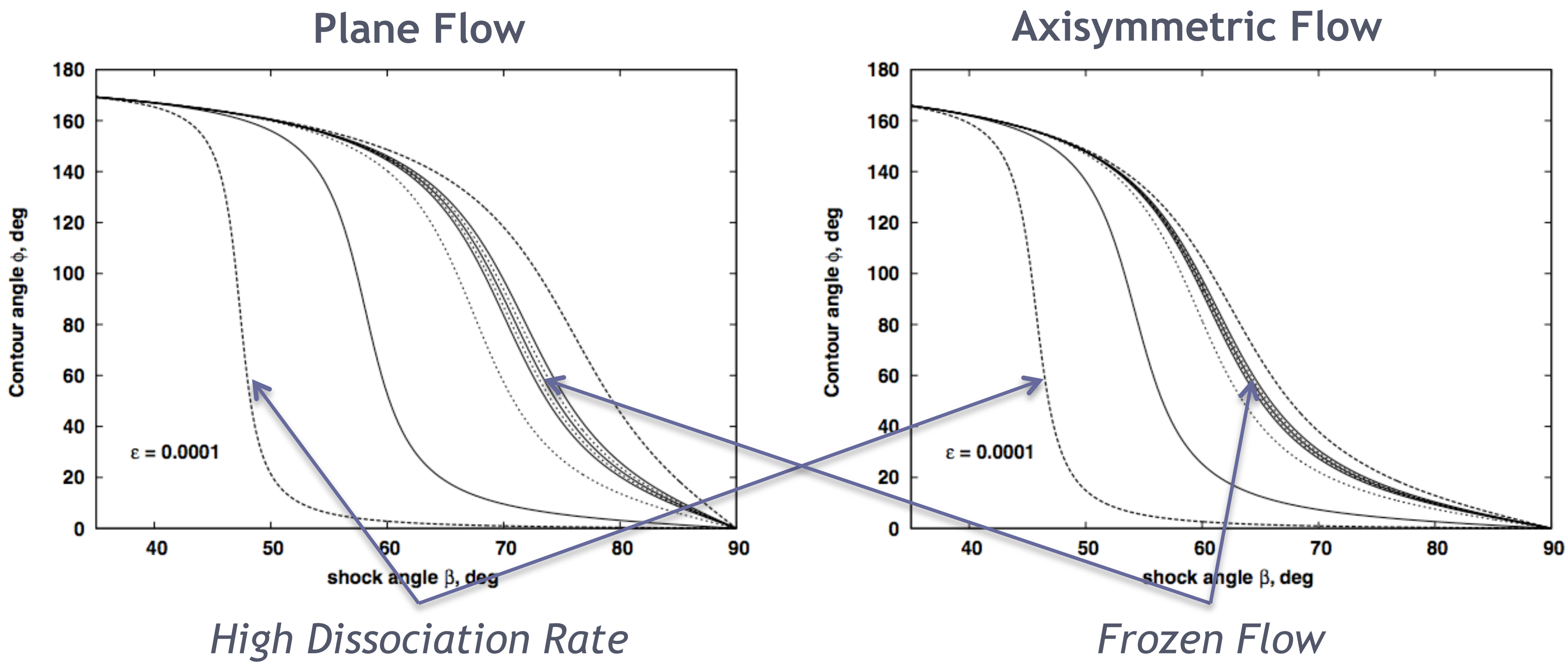
Density Field



High Dissociation Rate *Frozen Flow*

- At small shock angles, *β*, the contour angle is approximately 180°.
- As *β* increases, suddenly the contour angle changes to 0°.
- In the small range where ϕ changes rapidly, there is a saddle point.
- As dissociation rate increases, the "switch" angle decreases greatly.

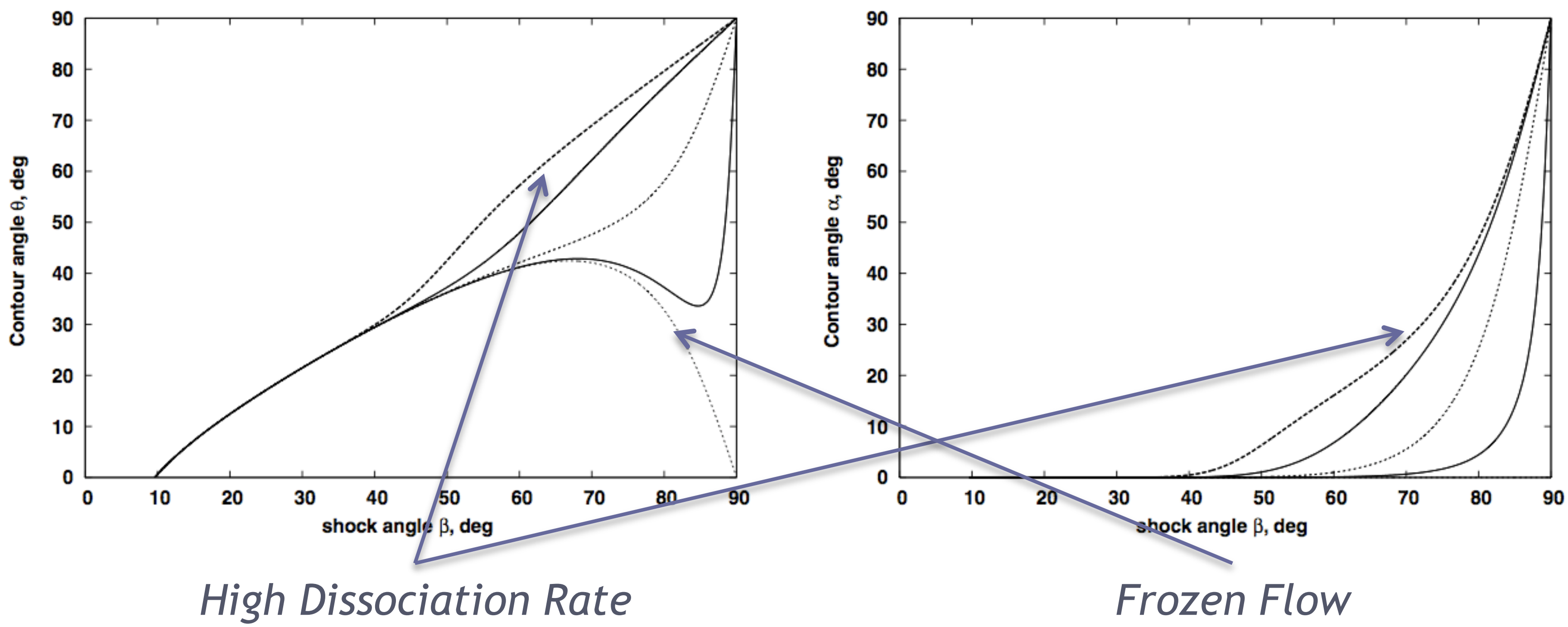
Pressure Field



- Similar to density, there is a saddle point at the shock.
- Much less abrupt though
- Less sensitive to the reaction rate.
- For exothermic reaction, almost no change in pressure contour angle.

Entropy Field

Contour vs. Free Stream Direction | **Contour vs. Streamline Direction**



- The entropy increases in flow direction along a streamline.
- Endothermic and Exothermic have same effect on contours.
- Near stagnation point, entropy contour is normal to streamline, but always asymptotically approaches streamline as flows reach equilibrium.

Questions?