Errata and Suggested Updates for

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1. In many places "non-linear" appears that should be recast as "nonlinear." This is found on pages 248, 559, 559, 566, 566, 566.

2. p. 98: Sec. 2.6.3: In the analysis of Eqs. (2.346-2.348), several factors of 1/2 are missing; e.g. dx_1 should be replaced by $(dx_1)/2$ in Eq. (2.346).

3. p. 89, 91, 112, 113: We would be better to employ an opposite sign convention for torsion τ. This is more common in the literature. For example, the Frenet-Serret equations, Eqs. (2.259-2.261), p. 89, are better stated with the more common sign convection as

$$\begin{aligned} \frac{d\mathbf{t}}{ds} &= \kappa\mathbf{n}, \\ \frac{d\mathbf{n}}{ds} &= -\kappa\mathbf{t} + \tau\mathbf{b}, \\ \frac{d\mathbf{b}}{ds} &= -\tau\mathbf{n}. \end{aligned}$$

4. p. 98: Note: some sources define

$$\text{div } \mathbf{T} = \nabla \cdot \mathbf{T}^T = \frac{\partial T_{ji}}{\partial x_i}.$$

As long as the analysis is internally consistent, as it is here, it is correct.

5. p. 111: Problem 4g is a repeat of Problem 1.

6. p. 113: Problem 22–The first equation in the problem may need to be

$$\mathbf{t}^T \cdot \frac{d^2\mathbf{t}}{ds^2} \times \frac{d\mathbf{t}}{ds} = \kappa^2\tau$$

This would change if we change the sign convention for τ.

7. p. 113: Problem 30 should have corners at $(0,0,0)$, $(0,1,0)$, $(1,1,0)$, and $(1,0,0)$.

8. p. 113: Problem 30 should have "For $\mathbf{f}(x,y,z) =$...."

9. p. 114: Problem 38 should read "...one finds that the three principal..."

10. p. 114: Problem 38 should have $q_i = -k_{ij}\partial T/\partial x_j$....

11. p. 123-124: Much of the text from Eq. (3.55) to Eq. (3.65) is better replaced by the following, removing the need for F:

    The general first-order linear equation

$$\frac{dy(x)}{dx} + P(x)y(x) = Q(x),$$

with

$$y(x_0) = y_0,$$

can be solved using the integrating factor $e^{\int_{x_0}^{x} P(s)\ ds}$. Multiply by the integrating factor and proceed:

$$\left(e^{\int_{x_0}^{x} P(s)\ ds}\right)\frac{dy(x)}{dx} + \left(e^{\int_{x_0}^{x} P(s)\ ds}\right)P(x)\ y(x) = \left(e^{\int_{x_0}^{x} P(s)\ ds}\right)Q(x).$$

Now use the product rule to combine terms on the left side:

$$\frac{d}{dx}\left(e^{\int_{x_0}^{x} P(s)\ ds}y(x)\right) = \left(e^{\int_{x_0}^{x} P(s)\ ds}\right)Q(x).$$

Next replace x by t

$$\frac{d}{dt}\left(e^{\int_{x_0}^{t} P(s)\ ds}y(t)\right) = \left(e^{\int_{x_0}^{t} P(s)\ ds}\right)Q(t),$$

Now apply the operator $\int_{x_0}^{x}(\cdot)\ dt$ to both sides:

$$\begin{aligned}
\int_{x_0}^{x}\frac{d}{dt}\left(e^{\int_{x_0}^{t} P(s)\ ds}y(t)\right)dt &= \int_{x_0}^{x}\left(e^{\int_{x_0}^{t} P(s)\ ds}\right)Q(t)\ dt,\\
e^{\int_{x_0}^{x} P(s)\ ds}y(x) - e^{\int_{x_0}^{x_0} P(s)\ ds}y(x_0) &= \int_{x_0}^{x}\left(e^{\int_{x_0}^{t} P(s)\ ds}\right)Q(t)\ dt,\\
e^{\int_{x_0}^{x} P(s)\ ds}y(x) - y(x_0) &= \int_{x_0}^{x}\left(e^{\int_{x_0}^{t} P(s)\ ds}\right)Q(t)\ dt,
\end{aligned}$$

which yields an exact solution for $y(x)$ in terms of arbitrary $P(x)$, $Q(x)$, x_0, and y_0:

$$y(x) = e^{-\int_{x_0}^{x} P(s)\ ds}\left(y_0 + \int_{x_0}^{x}\left(e^{\int_{x_0}^{t} P(s)\ ds}\right)Q(t)\ dt\right).$$

12. p. 124: Example 3.7 should be modified slightly. Replace a by $x_0 = 0$ in Eq. (3.69) and (3.70). Remove Eq. (3.72). Remove x_0 in Eq. (3.77).

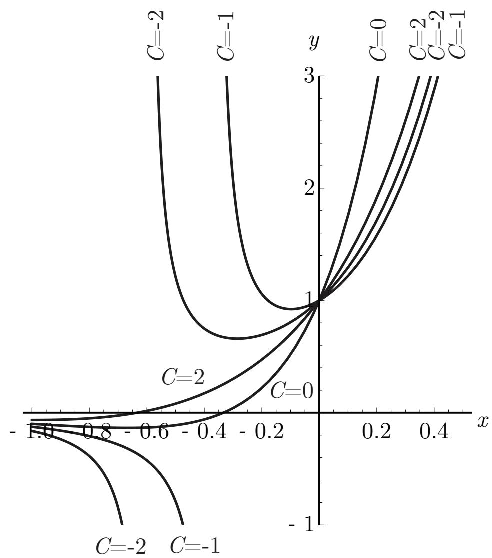

Figure 1: Modified Fig. 3.10. $y(x)$ which solves $y' = \exp(-3x)/x - y/x + 3\exp(3x)$.

13. p. 128: Figure 3.10 has a few incorrect labels for C in the first quadrant. The solution for various values of C is plotted in Fig. 3.10. Also insert the text

    "Solving for C, we get

    $$C = -\frac{x}{e^{3x} - y} - \frac{e^{-3x}}{3}.$$

    There is a singularity when $y = e^{3x}$."

14. p. 145: Problem 29: The problem is better stated as "...are possible when $b = 2.2617$, and find the corresponding frequency."

15. p. 156: Eq. (4.102) has a small error. The argument of the second term should be $-i\sqrt{14}\ln x$. So we should have

    $$\begin{aligned} y(x) &= \frac{1}{x}\left(C_1(\exp(\ln x))^{i\sqrt{14}} + C_2(\exp(\ln x))^{-i\sqrt{14}}\right), \\ &= \frac{1}{x}\left(C_1\exp(i\sqrt{14}\ln x) + C_2\exp(-i\sqrt{14}\ln x)\right), \\ &= \frac{1}{x}\left(\hat{C}_1\cos(\sqrt{14}\ln x) + \hat{C}_2\sin(\sqrt{14}\ln x)\right). \end{aligned}$$

16. p.181: The sentence after Eq. (4.380)

"One can also show the Legendre functions of the second kind, $Q_n(x)$, satisfy a similar orthogonality condition."

is better stated as

"The orthogonality condition for the Legendre functions of the second kind, $Q_n(x)$, is not straightforward."

A similar comment can be made for the Hermite and Laguerre polynomials, $\hat{H}_n(x)$, $\hat{L}_n(x)$, as well as the Bessel functions of the second kind $Y_\nu(\mu x)$.

17. p. 184: An additional sentence of clarification should appear at the bottom of the page:

    "Some sources define the complementary functions differently, taking them to be $U_n(x) = V_{n+1}(x)/\sqrt{1-x^2}$. The difference is not substantive."

18. p. 214: Problem 29: We should have

$$P(u,v) = \sum_{k=1}^{n}\sum_{j=1}^{k}(-1)^{j-1}\frac{d^{k-j}u}{dx^{k-j}}\ \frac{d^{j-1}}{dx^{j-1}}(a_{n-k}v).$$

19. p. 216: Problem 4.52, should read "...to being connected as well as identical to mass 1..." and "...of the two masses and the potential energy of the three springs."

20. p. 217: Problem 4.53 needs an additional factor of "2" in the error function, so as to be consistent with the correct Eq. (A.103) on p. 594.

21. p. 230: Fig. 5.5's caption should be $dy/dx = -\sqrt{x}y$

22. p. 236: The second term in Eq. (5.146) should be

$$2k\left(1 + x + \frac{1}{4}x^2 + ...\right)$$

    instead of

$$2k\left(1 + x + \frac{1}{2}x^2 + ...\right)$$

23. p. 237. It would be useful to add the following text and figure at the end of Example 5.10:

    "Detailed calculation reveals that y_1 and y_2 can be expressed as

$$\begin{aligned} y_1(x) &= \sqrt{x}I_1(2\sqrt{x}), \\ y_2(x) &= (1-2\gamma)\sqrt{x}I_1(2\sqrt{x}) + 2\sqrt{x}K_1(2\sqrt{x}). \end{aligned}$$

    Here γ is known as Euler's constant. It is given by

$$\gamma = \lim_{n\to\infty}\left(-\ln n + \sum_{m=1}^{n}\frac{1}{m}\right) = 0.5772156649...$$

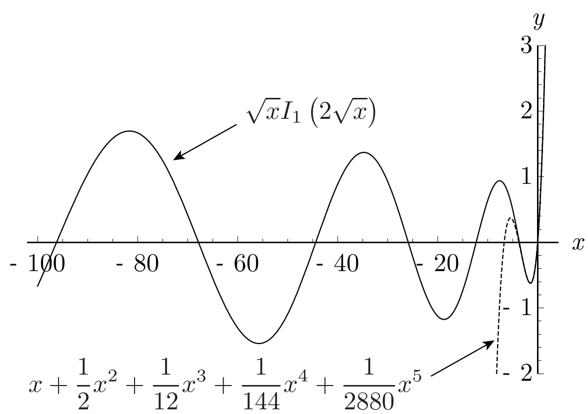

Figure 2: Exact and five term series solutions of $xy'' - y = 0$ with $y(0) = 0$, $y'(0) = 1$.

It has not been proved to be irrational.

If $y(0) = 0$, $y'(0) = 1$, the solution is

$$y = \sqrt{x}I_1\left(2\sqrt{x}\right).$$

The Taylor series representation of the solution is

$$y = x + \frac{1}{2}x^2 + \frac{1}{12}x^3 + \frac{1}{144}x^4 + \frac{1}{2880}x^5 + \ldots.$$

Clearly the Taylor series representation is Eq. (5.151) with $C_1 = 1$, and $C_2 = 0$. The exact and five term series solutions are plotted in Fig. 2. For $x < 0$, the exact equation predicts a locally oscillatory solution. The wavelength of the oscillation increases as $x \to -\infty$. For $x > 0$, the exact equation predicts a locally exponential solution. The series solution is accurate only for a limited range, $x \in [-4, 10]$."

24. p. 238: Just after Eq. (5.159), replace the phrase "..and the linear approximation to the exact solution..." with "...and a two-term series approximation to the exact solution..."

25. p. 239: Eq. (5.179) should be

$$y = x + \frac{x^2}{2} + \frac{x^4}{12} + \ldots.$$

26. p. 257: Eq. (5.319) should have

$$y(x) = y_0(x) + \epsilon y_1(x) + \epsilon^2 y_2(x)\ldots.$$

27. p. 269: Just before Eq. (5.429) we should find $\xi^2 + \eta^2 = r^2$ instead of $x^2 + y^2 = r^2$.

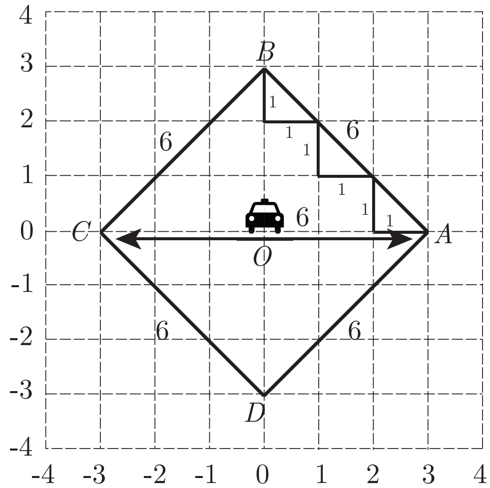

Figure 3: Circle of diameter 6 and circumference 24 in an $\ell_1(\mathbb{R}^2)$ Banach space.

28. p. 272: Problem 15 is identical to Example 5.11.
29. p. 276: Problem 55 repeats Problem 50.
30. p. 277: Problem 57 repeats Problem 18.
31. p. 292: The following example would be a useful addition.

    For $x \in \ell_1(\mathbb{R}^2)$ find the numerical value of π, defined as the ratio of the circumference to the diameter of a circle in this space.

    For $\ell_1(\mathbb{R}^2)$, we give a sketch of a circle of diameter 6 in Fig. 3. In this space our mathematical taxicab, depicted at the origin $O : (0, 0)$, can only move horizontally or vertically. Its distance to either points A, B, C, or D is 3 units. And combinations of horizontal and vertical motions that sum to 3 units can also take the taxicab to anywhere on line segments AB, BC, CD, or DA. So each point on $ABCD$ is three units from the origin. This represents a circle of radius 3. Thus, its diameter $\mathcal{D}$ is 6. Now in a Euclidean space we would have $|AB| = \sqrt{3^2 + 3^2} = 3\sqrt{2} = 4.243$. But one literally cannot cut corners in $\ell_1(\mathbb{R}^2)$. So to drive from A to B requires traversing a series of horizontal and vertical segments, and their distances add to 6. So each side of $ABCD$ has length 6, giving the circumference $\mathcal{C}$ of this circle as $4(6) = 24$. Thus our estimate for π in this space is

$$\pi_{\ell_1(\mathbb{R}^2)} = \frac{\mathcal{C}}{\mathcal{D}} = \frac{24}{6} = 4.$$

    More formally, we can say in this space the distance d from any point

$P : (x_1, x_2)$ to the origin is

$$d = |OP| = |x_1| + |x_2|.$$

Thus for us $|OA| = |OB| = |OC| = |OD| = 3$, the radius of the circle. And the distance from a point $P^{(1)} : (x_1^{(1)}, x_2^{(1)})$ to $P^{(2)} : (x_1^{(2)}, x_2^{(2)})$ is

$$d = |P^{(1)}P^{(2)}| = |x_1^{(1)} - x_1^{(2)}| + |x_2^{(1)} - x_2^{(2)}|.$$

So $|AB|$ is

$$|AB| = |3 - 0| + |0 - 3| = 6.$$

If $|OA| = a$, $|OB| = b$, and $|AB| = c$, we have an analog to the Pythagorean theorem for the triangle OAB:

$$c = a + b.$$

If we were in a Euclidian space, $\ell_2(\mathbb{R}^2)$, we would have the traditional $c^2 = a^2 + b^2$.

32. p. 296: The equation appearing just before Eq. (6.92) should receive a number.

33. p. 302: In Example 6.19, $||x||_2 = 3.873$ not 3.870.

34. p. 311: In Fig. 6.9, need more space in needed in the term "$-0.23 \sin 4t$."

35. p. 316: Eq. (6.239) needs an approximate equals sign rather than an equals sign. So we should find

$$t^2 \approx \sum_{n=1}^{N} \alpha_n \varphi_n.$$

36. p. 319. Eqs. (6.264, 6.265) each have the same small error. The associated figures are correct. We should find

"Taking this into account and retaining only the necessary basis functions, we can write the Fourier sine series as

$$x(t) = t(1 - t) \sim x_p(t) = \sum_{m=1}^{N} \frac{8}{(2m-1)^3\pi^3} \sin(2m-1)\pi t.$$

The norm of the error is then

$$||x(t) - x_p(t)||_2 = \sqrt{\int_0^1 \left(t(1-t) - \left(\sum_{m=1}^{N} \frac{8}{(2m-1)^3\pi^3} \sin(2m-1)\pi t \right) \right)^2 dt}.$$

"

37. p. 327: Eq. (6.335) has a $\cdot$ between the matrix and the vector and should not. We should find

$$\begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x^1 \\ x^2 \end{pmatrix}.$$

38. p. 335: Eq. (6.401) has an extra comma that should be removed in a_{21},

39. p. 349: Within Example 6.53, the analysis of Eq. (6.563) and below should be restructured to be consistent with the definition of the adjoint in Eq. (6.386): $\langle \mathbf{L}x, y\rangle = \langle x, \mathbf{L}^* y\rangle$. So very small changes are useful in Eqs. (6.563-6.566). The results are not changed in any substantive way. Thus, the text is better as

"Now, because by definition $\langle \mathbf{L}x_n, y_m\rangle = \langle x_n, \mathbf{L}^* y_m\rangle$, we have

$$\begin{aligned} \langle \mathbf{L}x_n, y_m\rangle - \langle x_n, \mathbf{L}^* y_m\rangle &= 0, \\ \langle \lambda_n x_n, y_m\rangle - \langle x_n, \lambda_m^* y_m\rangle &= 0, \\ \overline{\lambda_n}\langle x_n, y_m\rangle - \lambda_m^*\langle x_n, y_m\rangle &= 0, \\ (\lambda_n^* - \lambda_m^*)\langle x_n, y_m\rangle &= 0. \end{aligned}$$

So, for $m = n$, we get $\langle x_n, y_n\rangle \neq 0$, and for $m \neq n$, we get $\langle x_n, y_m\rangle = 0$. Thus, we must have the so-called bi-orthogonality condition

$$\langle x_n, y_m\rangle = D_{nm},$$

$$D_{nm} = 0 \qquad \text{if} \qquad m \neq n.$$

Here D_{nm} is a diagonal matrix that can be reduced to the identity matrix with proper normalization. Also because x_n and y_m are strictly real, the inner product commutes and we have

$$\langle y_m, x_n\rangle = \langle x_n, y_m\rangle = D_{nm}.$$

Now consider the following series of operations on the original form of the expansion we seek

$$\begin{aligned} f(s) &= \sum_{n=1}^{N} \alpha_n x_n(s), \\ \langle y_m(s), f(s)\rangle &= \langle y_m(s), \sum_{n=1}^{N} \alpha_n x_n(s)\rangle, \\ \langle y_m(s), f(s)\rangle &= \sum_{n=1}^{N} \alpha_n \langle y_m(s), x_n(s)\rangle, \\ \langle y_m(s), f(s)\rangle &= \alpha_m \langle y_m(s), x_m(s)\rangle, \\ \alpha_m &= \frac{\langle y_m(s), f(s)\rangle}{\langle y_m(s), x_m(s)\rangle}, \\ \alpha_n &= \frac{\langle y_n(s), f(s)\rangle}{\langle y_n(s), x_n(s)\rangle}, \qquad n = 1, 2, 3, \ldots \end{aligned}$$

Now in the case at hand, ...”

40. p. 363: Eq. (6.693) should have $\alpha\phi(t)$ instead of $c\phi(t)$.
41. pp. 364-5: In Eqs. (6.702), (6.703), one could replace $(t^{5/4})^{2/5}$ by $t^{1/2}$. Additional simplification is possible that is easily achieved with computer algebra.
42. p. 371: Eq. (6.754) should have λt^3 instead of λt^2. So we should find

$$\begin{aligned} \int_0^1 \left(-(1-2t)^2 + \lambda t^3 (1-t)^2\right) dt &= 0, \\ -\frac{1}{3} + \frac{\lambda}{60} &= 0, \\ \lambda &= 20. \end{aligned}$$

43. p. 378: The y-axis of the rightmost graph of Fig. 6.28 is mis-labeled. It should be $y(t=1)$.
44. p. 379: Problem 10 is too similar to Example 6.39.
45. p. 381: Problem 27: the inner product should be enclosed by angle brackets instead of parentheses. So we should have

$$\langle x, y\rangle = \int_a^b w(t)x(t)y(t)\ dt,$$

46. p. 381: Problem 30 is a repeat of Problem 18.
47. p. 381: Problem 31 is a repeat of Problem 8.
48. p. 381: Problem 33 is a repeat of Problem 4.
49. p. 382: Problem 37 is a repeat of Problem 11.
50. p. 384: Problem 57 is a repeat of Problem 14.
51. p. 385: Problem 61 is a repeat of Problem 56.
52. p. 385: Problem 66 is a repeat of Problem 64.
53. pp. 385-386: Problem 70 should have $r(t)$ instead of $r(x)$.
54. p. 386: Problem 72 may benefit from changing the lower limit to $-\infty$.

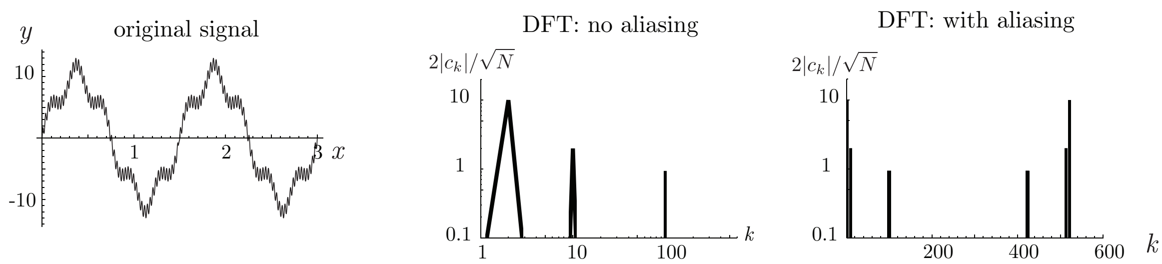

Figure 4: Modified Fig. 7.5.

55. p. 424: Eq. (7.229) should actually employ $\mathbf{u}\mathbf{u}^T$; it presently incorrectly uses $\mathbf{u}^T \cdot \mathbf{u}$, though Eq. (7.230) is correct. We should thus find

$$\begin{aligned}
\mathbf{H} &= \mathbf{I} - 2\mathbf{u}\mathbf{u}^T, \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 2 \begin{pmatrix} -\frac{1}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{pmatrix}, \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 2 \begin{pmatrix} \frac{1}{10} & \frac{-3}{10} \\ \frac{-3}{10} & \frac{9}{10} \end{pmatrix}, \\
&= \begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{-4}{5} \end{pmatrix}.
\end{aligned}$$

56. p. 426: Eq. (7.240) should read $\mathbf{I} - 2\mathbf{u}\mathbf{u}^T$. The actual numerical values are correct.

57. p. 431: Figs. 7.5 and 7.6 would be more effective if it plotted $2|c_k|/\sqrt{N}$ vs. k. When so done, it returns the actual amplitude of the mode for modes that are sufficiently resolved. It would be useful to present the analysis to show this. Figure 4 gives an example of how this could be fixed.

58. p. 439: To illustrate an application of the $\mathbf{Q} \cdot \mathbf{U}$ decomposition, it would be useful to use it to find the roots of a polynomial via identification of the eigenvalues of the so-called “companion matrix.” An iterative method involving the $\mathbf{Q} \cdot \mathbf{U}$ decomposition may be employed.

59. p. 440: Eq. (7.337) has an unneeded dot in the matrix product.

60. p. 441: It would be useful in Section 7.9.5 to have more discussion of algebraic and geometric multiplicity of the eigenspace.

61. p. 450: Columns of $\mathbf{Q}$ should be the normalized eigenvectors, not just the eigenvectors.

62. p. 457: More nuance is needed for the projection matrix $\mathbf{P}$. The matrix $\mathbf{P}$ as defined by Eq. (7.477) is guaranteed symmetric with spectral norm

of 1 with eigenvalues of 1 or 0. And it guarantees $\mathbf{P} \cdot \mathbf{x} = \mathbf{P} \cdot \mathbf{P} \cdot \mathbf{x}$, as required. However if

$$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix},$$

we find a) $\mathbf{B} \cdot \mathbf{x} = \mathbf{B} \cdot \mathbf{B} \cdot \mathbf{x}$, b) the eigenvalues of $\mathbf{B}$ are 1 and 0; c) $\mathbf{B}$ is asymmetric, d) the spectral norm of $\mathbf{B}$ is $||\mathbf{B}|| = \sqrt{2}$, thus $\mathbf{B}$ can stretch vectors of certain orientation, e.g. if $\mathbf{x} = (1,0)^T$, $\mathbf{B} \cdot \mathbf{x} = (1,1)^T$. In fact here $\mathbf{B}$ should be called a *non-orthogonal projection* or an *oblique projection*. Oblique projections satisfy $\mathbf{B} \cdot \mathbf{B} = \mathbf{B}$ but are not symmetric: $\mathbf{B} \neq \mathbf{B}^T$. In contrast *orthogonal projection* matrices $\mathbf{P}$ satisfy $\mathbf{P} \cdot \mathbf{P} = \mathbf{P}$ and $\mathbf{P} = \mathbf{P}^T$ with spectral norm of 1.

63. p. 460: Eq. (7.497) needs another "dot" within its matrix multiplication. So we should find

$$\mathbf{a} = \left((\mathbf{W} \cdot \mathbf{A})^T \cdot \mathbf{W} \cdot \mathbf{A} \right)^{-1} \cdot (\mathbf{W} \cdot \mathbf{A})^T \cdot \mathbf{W} \cdot \mathbf{b}.$$

64. p. 469: In Eq. (7.593), we should have $q_{11} = 1/\sqrt{10}$.

65. p. 475: Problem 32 has no Cholesky decomposition as the matrix is not positive definite.

66. p. 486: Figure 8.2 needs negative signs on some numbers on the y axis.

67. p. 491: Some confusion exists here. The ϕ_i of Eq. (8.72) is orthonormal. The ϕ_i of Eq. (8.73) is not. It is written carefully, but it is confusing. It could and should be reformulated for more clarity.

68. p. 506: Eq. (9.60) needs a small correction. It should have

$$\mathbf{S}^{-1} = \begin{pmatrix} \frac{-i}{\sqrt{3}} & \frac{1}{2} + \frac{\sqrt{3}}{6} i \\ \frac{i}{\sqrt{3}} & \frac{1}{2} - \frac{\sqrt{3}}{6} i \end{pmatrix}$$

69. p. 518: Just after Eq. (9.159), should read "...eigenvectors $\mathbf{e}_k$, $k = 1, 2, ..., K$, as possible."

70. p. 553: In the caption of Fig. 9.13, one should replace dx/dx by dx/dt.

71. p. 562: Eq. (9.494) should have an 8 in the numerator of both terms, not $4\sqrt{2}$. So we should find

$$T(x,t) \approx \frac{8}{\pi^3} e^{-\pi^2 t} \sin(\pi x) + \frac{8}{27\pi^3} e^{-9\pi^2 t} \sin(3\pi x).$$

72. p. 562: Eq. (9.495) should have additional $\sqrt{2}$ on both terms because this is part of the basis function. Thus, we should find

$$T(x,t) \approx \alpha_1(t) \sqrt{2} \sin(\pi x) + \alpha_2(t) \sqrt{2} \sin(3\pi x),$$