AME 561

Examination 1

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1. (20) Find the point on the surface $x + y + z = 1$ that is closest to the point $(x, y, z) = (1, 2, 3)$.

2. (20) Find a solution which satisfies the differential equation and boundary conditions:

$$\frac{dy}{dx}\frac{d^2y}{dx^2} + y = 1, \qquad \left.\frac{dy}{dx}\right|_{x=1} = 0, \qquad y(1) = 0.$$

3. (20) Find a power series solution to the differential equation with boundary conditions

$$\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 0, \qquad y(0) = 1, \qquad \left.\frac{dy}{dx}\right|_{x=0} = 0.$$

4. (20) For $0 < \epsilon << 1$, find all solutions valid at $O(\epsilon)$ to the system of equations

$$x + \epsilon y(x + 1) = e^\epsilon, \qquad y^2 + \frac{xy}{1 + \epsilon x} = 2.$$

5. (20) Find a solution $y(x)$ for arbitrary $f(x)$ for the following differential equation and initial conditions using the Green's function technique:

$$\frac{d^2y}{dx^2} = f(x); \qquad y(0) = 0, \qquad \left.\frac{dy}{dx}\right|_{x=1} = 0.$$

Take as your domain $0 < x < 1$.