AME 561
Examination 2
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1. (20) Given $x \in \mathcal{R}^1$, $f : \mathcal{R}^1 \to \mathcal{R}^1$,

$$f(x) = \begin{cases} 0, & x \in [-2, 2) \\ 1, & x \in [2, 3]. \end{cases}$$

find the first two terms in a Fourier-sine expansion of $f(x)$.

2. (20) Show, using Cartesian index notation, that

$$\frac{1}{2}\nabla(\mathbf{u} \cdot \mathbf{u}) = (\mathbf{u} \cdot \nabla)\mathbf{u} + \mathbf{u} \times (\nabla \times \mathbf{u}).$$

3. (20) For $\mathbf{A} : \mathcal{C}^2 \to \mathcal{C}^3$, find the vector $\mathbf{x} \in \mathcal{C}^2$ of minimum $||\mathbf{x}||_2$ which minimizes $||\mathbf{A} \cdot \mathbf{x} - \mathbf{b}||_2$ when

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ i & 2i \\ 1 & 0 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} i \\ 0 \\ 0 \end{pmatrix}.$$

4. (20) Consider the vector space in which vectors $\mathbf{X}$ and $\mathbf{Y}$ are complex two-by-two matrices, that is $\mathbf{X}, \mathbf{Y} \in \mathcal{C}^{2\times 2}$ with

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}, \qquad \mathbf{Y} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix}.$$

Determine if the operation $\mathbf{X} : \mathbf{Y} = \sum_{i=1}^{2} \sum_{j=1}^{2} \overline{x}_{ij} y_{ij}$ is an inner product.

5. (20) Find a set of reciprocal basis vectors in $\mathcal{R}^2$ to the basis vectors

$$u_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \qquad u_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$