AME 561
Examination 1
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1. (25) Find $y(x)$ which renders the the integral

$$\int_0^1 \left(\frac{dy}{dx}\right)^2 \, dx$$

to be an extremum subject to

$$y(0) = 0, \quad y(1) = 1, \qquad \text{and} \qquad \int_0^1 y \, dx = 1.$$

2. (25) Find a solution which satisfies the differential equation and boundary conditions:

$$\frac{d^2y}{dx^2} - 2y\frac{dy}{dx} = 0, \qquad \left.\frac{dy}{dx}\right|_{x=0} = 3, \qquad y(0) = 0.$$

Sketch the solution.

3. (25) Find the most general solution to

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}.$$

4. (25) For $0 < \epsilon << 1$, find a solution, uniformly valid throughout the domain $x \in [0, 1]$, to

$$\epsilon\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{x}{y} = 0, \qquad y(0) = 1, \quad y(1) = 1.$$

Sketch the solution.