AME 561
Examination 2
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1. (15) Find the curvature of the parabola $y = x^2$ at the point $x = 1$.

2. (15) Find the matrix $\mathbf{A}$ that operates on any vector in the $x - y$ plane so as to turn it through a counterclockwise angle θ about the z-axis without changing its length.

3. (20) Given $x \in \mathbb{R}^1, f : \mathbb{R}^1 \to \mathbb{R}^1$,

$$f(x) = \frac{1}{x}, \qquad x \in [1, 3],$$

find the first term in a Fourier-Laguerre expansion of $f(x)$. The set of orthonormal functions which arise from the Laguerre equation are $\varphi_n(s) = \left\{e^{-s/2}, e^{-s/2}(1-s), \ldots, e^{-s/2}L_n(s)\right\}$. It is acceptable to express your answer in terms of a definite integral.

4. (20) For $x \in [0, 1] \in \mathbb{R}^1, y \in \mathbb{L}_2[0, 1]$, consider

$$\frac{d^2y}{dx^2} + 8\sqrt{y} = x, \qquad y(0) = 0, \qquad y(1) = 0.$$

Use a one term collocation method to find an approximate solution.

5. (30) Consider

$$\mathbf{A} = \begin{pmatrix} 1 & i \\ 0 & 0 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

   (a) Find $||\mathbf{A}||_2$.

   (b) Find the most general $\mathbf{x}$ which minimizes $||\mathbf{A} \cdot \mathbf{x} - \mathbf{b}||_2$.

   (c) Of all the vectors which minimize $||\mathbf{A} \cdot \mathbf{x} - \mathbf{b}||_2$, find the vector $\mathbf{x}$ with minimum $||\mathbf{x}||_2$.