AME 60611
Examination 2
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1. (25) Consider the curve in $\mathbb{R}^3$ defined parametrically by

$$\begin{aligned} x &= t, \\ y &= t, \\ z &= t^2. \end{aligned}$$

   (a) Find the length of the curve from $(0,0,0)$ to $(1,1,1)$. You need not numerically evaluate the resulting integral.

   (b) Find the unit tangent at the point $(1,1,1)$.

2. (25) Consider two functions in $\mathbb{L}_2[0,1]$: $v_1 = 1$, $v_2 = t^3$.

   (a) Determine if $v_1$ and $v_2$ are orthonormal.

   (b) Project the Heaviside function $H(t-1/2)$ onto the space spanned by $v_1$ and $v_2$; that is, find the constants $\alpha_1$, $\alpha_2$ that best approximate

$$\alpha_1 v_1 + \alpha_2 v_2 \sim H(t-1/2).$$

3. (25) For $\mathbf{A} : \mathbb{C}^3 \to \mathbb{C}^2$, find the vector $\mathbf{x} \in \mathbb{C}^3$ of smallest $||\mathbf{x}||_2$ which minimizes the error norm $||\mathbf{A} \cdot \mathbf{x} - \mathbf{b}||_2$ when

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & i \\ 2 & 2 & 2i \end{pmatrix}.$$

   and

$$\mathbf{b} = \begin{pmatrix} 0 \\ 1+i \end{pmatrix}.$$

4. (25) Consider

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}.$$

   Cast the matrix $\mathbf{A}$ into Jordan canonical form; that is, find matrices $\mathbf{S}$ and $\mathbf{J}$ such that $\mathbf{A} = \mathbf{S} \cdot \mathbf{J} \cdot \mathbf{S}^{-1}$.