AME 60611
Examination 1
J. M. Powers
8 October 2007

1. (20) Find all $y(x)$ which satisfy
$$\frac{d^4y}{dx^4} - y = x^2 + e^x.$$

2. (20) Use the method of strained coordinates to find the appropriate frequency modulation, valid at order ϵ, to achieve a secularity-free solution to the equation
$$\frac{d^2x}{dt^2} + x + \epsilon x^5 = 0, \qquad x(0) = 0, \left.\frac{dx}{dt}\right|_{t=0} = 1.$$
You have the identities
$$\sin^5\theta = \frac{5}{8}\sin\theta - \frac{5}{16}\sin 3\theta + \frac{1}{16}\sin 5\theta; \;\; \cos^5\theta = \frac{5}{8}\cos\theta + \frac{5}{16}\cos 3\theta + \frac{1}{16}\cos 5\theta.$$

3. (20) Find $\left.\frac{\partial u}{\partial x}\right|_y$ if
$$\begin{aligned} x + 2y + \sin u \sin v &= 1, \\ u^2 + v^2 &= xy. \end{aligned}$$

4. (20) Find all $y(x)$ which satisfy
$$\left(\frac{d^2y}{dx^2}\right)^2 + x\frac{d^2y}{dx^2} = \frac{dy}{dx}, \quad y'(0) = 1, \; y(0) = 0.$$

5. (20) Consider the transformation from non-Cartesian coordinates (x^1, x^2) to Cartesian coordinates (ξ^1, ξ^2):
$$\begin{aligned} \xi^1 &= (x^1)^2, \\ \xi^2 &= x^1 + 2x^2. \end{aligned}$$
A vector field $\mathbf{u}$ has Cartesian representation $U^i = (2\xi^1, 3\xi^2)^T$. Find
   (a) the metric tensor of the transformation, and
   (b) an expression for the vector field components u^i in the non-Cartesian system, (x^1, x^2).