AME 60611
Examination 2
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1. (20) Consider the lines in $\mathbb{E}^3$ given by

$$x = y = z$$

and

$$3x + y = y + 1 = z - 1.$$

It is straightforward to find the distance from a point on one line to a point on the other. Find the coordinates of the two points, one on each line, which minimizes this distance, and find the value of the distance.

2. (20) Find $\mathbf{x}$ of minimum $||\mathbf{x}||_2$ which minimizes $||\mathbf{A} \cdot \mathbf{x} - \mathbf{b}||_2$ when

$$\mathbf{A} = \begin{pmatrix} 1+i & i \\ 2+2i & 2i \\ 1 & 0 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

3. (20) In $\mathbb{R}^3$, a set of vectors is given as

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \qquad v_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

   (a) Determine if these form a basis in $\mathbb{R}^3$.
   (b) Find the reciprocal basis.

4. (20) Use a one-term Galerkin method with a polynomial basis function to estimate the solution to the differential equation

$$\frac{d^3y}{dx^3} + y = x, \qquad y(0) = 0, \; y(1) = 0, \; y'(0) = 0.$$

5. (20) Use Cartesian index notation to prove the identity

$$\nabla \times (\nabla \times \mathbf{u}) = \nabla(\nabla \cdot \mathbf{u}) - \nabla \cdot \nabla \mathbf{u}.$$