AME 60611

Examination 1

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1. (25) Find the minimum distance between the ellipse described by $x^2 + 4y^2 = 1$ and the line $x + y = 2$.

2. (25) Find the appropriate Green's function solution for the differential equation

$$\frac{d^3y}{dx^3} = f(x), \qquad y(0) = 0, \ y'(0) = 0, \ y''(0) = 0.$$

Test your method if $f(x) = 1$.

3. (25) Find an exact solution for $y(x)$ if

$$x^2 \frac{d^2y}{dx^2} + y = 0, \qquad y(1) = 0, \ y'(1) = 1.$$

4. (25) For $0 < \epsilon << 1$, find a uniformly valid approximate solution to $y(x)$ which satisfies the differential equation and boundary conditions

$$\epsilon \frac{d^2y}{dx^2} + \frac{dy}{dx} + xy^2 = 0, \quad y(0) = 1, \ y(1) = 1.$$