AME 60611
Examination 1
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2 October 2009

1. (25) If

$$u^2 + v^2 + x^2 + y^2 = 1, \qquad u + 2v^3 + 3x^3 + 4y^4 = 0,$$

find $\frac{\partial u}{\partial x}\big|_y$.

2. (25) Solve

$$\frac{d^3y}{dx^3} + x\frac{d^2y}{dx^2} = x, \qquad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

(You may find the error function, defined as $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-s^2}\, ds$, to be useful.)

3. (25) For $0 < \epsilon << 1$, use boundary layer methods to find a uniformly valid asymptotic solution to

$$\epsilon\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - y = 0, \qquad y(0) = 0,\ y(1) = 0.$$

4. (25) Find a general solution to

$$\frac{d^2y}{dx^2} + y = x^2 + e^x.$$