AME 60611
Examination 2
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1. (20) For $x \in [0, 3]$, find the first two terms in a Fourier-Legendre expansion of the Heaviside unit step function:
$$f(x) = H(x - 1).$$
The first two Legendre polynomials are $P_o(s) = 1$, $P_1(s) = s$, for $s \in [-1, 1]$.

2. (20) Given
$$\mathbf{A} = \begin{pmatrix} \alpha & \beta \\ \beta & \gamma \end{pmatrix},$$
where $\alpha \in \mathbb{R}^1$, $\beta \in \mathbb{R}^1$, $\gamma \in \mathbb{R}^1$ find the conditions on α, β, and γ which insure that $\mathbf{A}$ is positive definite.

3. (20) For $t \in [0, 1]$, find all approximate solutions available from a one-term Galerkin method applied to the differential equation and initial conditions
$$\frac{d^2y}{dt^2} + y^2 = 1, \qquad y(0) = 0, \quad \left.\frac{dy}{dt}\right|_{t=0} = 0.$$

4. (20) Find the $\mathbf{x}$ of minimum $||\mathbf{x}||_2$ which minimizes the quantity $||\mathbf{A} \cdot \mathbf{x} - \mathbf{b}||_2$ when
$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

5. (20) For a scalar field $\phi(x_i)$, use Cartesian index notation and prove the curl of the gradient of that scalar field is zero.