AME 60611
Examination 1
J. M. Powers
30 September 2011

1. (25) Consider the transformation relating Cartesian (ξ^1, ξ^2) to a new coordinate system, (x^1, x^2):

$$\begin{aligned} \xi^1 &= x^1, \\ \xi^2 &= -x^1 + x^2. \end{aligned}$$

   (a) Find the Jacobian matrix, the metric tensor, determine if the mapping is orthogonal, area- and orientation-preserving.

   (b) Sketch lines of constant x^1 and x^2 in the (ξ^1, ξ^2) plane.

   (c) For a known function, $\phi(\xi^1, \xi^2)$, find a representation for $\partial\phi/\partial x^1$ and $\partial\phi/\partial x^2$ using appropriate transformation rules.

2. (25) Consider on the domain $x \in [0, \infty)$ the differential equation and initial condtion

$$\epsilon \frac{dy}{dx} + y = f(x), \qquad y(0) = 0.$$

   (a) For any ϵ, large or small, use the Green's function method to find a solution of the form

$$y(x) = \int_0^\infty g(x, s) f(s) ds.$$

   (b) Find $y(x)$ via the Green's function for $f(x) = 1$ and show from direct expansion of the Green's function solution that $y(x) \to f(x) = 1$ as $\epsilon \to 0$.

   (c) Discuss the solution for $y(x)$ when $f(x) = 1$ and $0 < \epsilon \ll 1$ in the context of boundary layer theory.

3. (25) Find the most general solution to

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = x.$$

4. (25) Find the general solution to

$$\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} = 0.$$