AME 60611
Examination 2
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1. (20) Consider the lines in $\mathbb{E}^3$ given by

$$x = y = z$$

and

$$x + y = x - y + 1 = z - 1.$$

It is straightforward to find the distance from a point on one line to a point on the other. Find the coordinates of the point on each line which minimizes this distance, and find the value of the distance.

2. (25) Find $\mathbf{x}$ of minimum $||\mathbf{x}||_2$ which minimizes $||\mathbf{A} \cdot \mathbf{x} - \mathbf{b}||_2$ when

$$\mathbf{A} = \begin{pmatrix} 1 & i \\ 2 & 2i \\ 0 & 0 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

3. (25) In $\mathbb{L}_2[0,1]$, we have the linearly independent functions $u_1 = t$, $u_2 = t^2$. Project the function $f(t) = t^3$ onto the space spanned by u_1 and u_2; thus, find the best α_1 and α_2 to approximate $f(t) \simeq \alpha_1 u_1 + \alpha_2 u_2$.

4. (25) Use a one-term Galerkin method with a polynomial basis function to estimate the solution to the differential equation

$$\frac{d^3y}{dx^3} + y = x, \qquad y(0) = 0, \; y(1) = 0, \; y'(0) = 0.$$

5. (5) Use Cartesian index notation to prove the identity

$$\nabla^T \cdot (\nabla \times \mathbf{u}) = 0.$$