

AME 60611
Examination 1
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4 October 2013

1. (20) Find the point on the surface $x + y + z = 1$ that is closest to the point $(x, y, z) = (1, 2, 3)$.
2. (20) Find a solution which satisfies the differential equation and boundary conditions:

$$\frac{dy}{dx} \frac{d^2y}{dx^2} + y = 1, \quad \left. \frac{dy}{dx} \right|_{x=1} = 0, \quad y(1) = 0.$$

3. (20) Find a power series solution to the differential equation with boundary conditions

$$\frac{d^2y}{dx^2} + 2x \frac{dy}{dx} + y = 0, \quad y(0) = 1, \quad \left. \frac{dy}{dx} \right|_{x=0} = 0.$$

4. (20) For $0 < \epsilon \ll 1$, find all solutions valid at $O(\epsilon)$ to the system of equations

$$x + \epsilon y(x + 1) = e^\epsilon, \quad y^2 + \frac{xy}{1 + \epsilon x} = 2.$$

5. (20) Find a solution $y(x)$ for arbitrary $f(x)$ for the following differential equation and initial conditions using the Green's function technique:

$$\frac{d^2y}{dx^2} = f(x); \quad y(0) = 0, \quad \left. \frac{dy}{dx} \right|_{x=1} = 0.$$

Take as your domain $x \in [0, 1]$.