AME 60611
Examination 2
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1. (20) Given $x \in \mathbb{R}^1$, $f : \mathbb{R}^1 \to \mathbb{R}^1$,

$$f(x) = \sin x, \qquad x \in [-1, 1],$$

find the first two terms in a Fourier-Legendre expansion of $f(x)$. The first two Legendre polynomials are $P_o(s) = 1$, $P_1(s) = s$, $s \in [-1, 1]$.

2. (20) If $\mathbf{u} = x\mathbf{i} + y\mathbf{j}$, evaluate

$$I = \oint_C \mathbf{u}^T \cdot d\mathbf{r},$$

where C is a single loop around the closed contour described by the ellipse, $x^2 + 4y^2 = 1$.

3. (20) For $x \in \overline{\mathbb{L}}_2[0, 1]$, $y \in \overline{\mathbb{L}}_2[0, 1]$, $t \in \mathbb{R}^1$, find the angle between the two functions

$$x(t) = it,$$
$$y(t) = t.$$

4. (20) For $\mathbf{A} : \mathbb{R}^3 \to \mathbb{R}^2$, find the vector $\mathbf{x} \in \mathbb{R}^3$ of minimum $||\mathbf{x}||_2$ which minimizes $||\mathbf{A} \cdot \mathbf{x} - \mathbf{b}||_2$ when

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}.$$

5. (20) Find the singular value decomposition of $\mathbf{A}$ where

$$\mathbf{A} = \begin{pmatrix} 3i & 4 & 0 \end{pmatrix}.$$