AME 60611
Examination 1
J. M. Powers
2 October 2015

1. (25) Consider the curve $\mathcal{C}$ defined by the intersection of two surfaces: 1) the unit sphere
$$x^2 + y^2 + z^2 = 1,$$
and 2) the plane
$$x + y + z = 1.$$
Find the minimum value of y on $\mathcal{C}$ and the values of x and z on $\mathcal{C}$ where y takes on its minimum value.

2. (25) Consider
$$x\frac{dy}{dx} - y^2 + y = 0, \qquad y(0) = -1.$$
Determine a solution if a solution exists. If it exists, determine whether it is unique.

3. (25) Use the Green's function method to find the general solution on the domain $x \in [0, \infty)$ to
$$\frac{dy}{dx} + y = f(x), \qquad y(0) = 1.$$
It can help to transform y to a new dependent variable to render the boundary condition to be homogeneous.

4. (25) If $0 < \epsilon << 1$, $x \in [0, 1]$, find an appropriate $O(1)$ and $O(\epsilon)$ solution for
$$x\frac{dy}{dx} - \epsilon y = 0, \qquad y(1) = 1.$$
Compare to the exact solution.