AME 60611
Examination 2
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1. (20) Consider the curve in $\mathbb{R}^3$ defined parametrically by

$$\begin{aligned} x &= \sqrt{t}, \\ y &= t, \\ z &= t. \end{aligned}$$

   (a) Find the length of the curve from $(0,0,0)$ to $(1,1,1)$. You need not numerically evaluate the resulting integral.
   (b) Find the unit tangent at the point $(1,1,1)$.

2. (20) Consider two functions in $\mathbb{L}_2[0,1]$: $v_1 = 1$, $v_2 = t$.

   (a) Determine if v_1 and v_2 are orthonormal.
   (b) Project the Heaviside function $H(t-1/2)$ onto the space spanned by v_1 and v_2; that is, find the constants α_1, α_2 that best approximate

$$\alpha_1 v_1 + \alpha_2 v_2 \approx H(t - 1/2).$$

3. (20) For $\mathbf{A} : \mathbb{C}^3 \to \mathbb{C}^2$, find the vector $\mathbf{x} \in \mathbb{C}^3$ of smallest $||\mathbf{x}||_2$ which minimizes the error norm $||\mathbf{A} \cdot \mathbf{x} - \mathbf{b}||_2$ when

$$\mathbf{A} = \begin{pmatrix} i & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

   and

$$\mathbf{b} = \begin{pmatrix} i \\ 1 \end{pmatrix}.$$

4. (20) Find a singular value decomposition of the matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

5. (20) Using a collocation method within the method of weighted residuals, find a one-term approximation to the solution of the following problem:

$$\frac{d^2y}{dx^2} - y = -x^3, \qquad y(0) = y(1) = 0.$$

   Choose an appropriate polynomial trial function.