NAME:
AME 561
Examination 1
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1. (25) Given that

$$u^6 + uv + 1 = x + y + z,$$
$$u^3 + uv = xyzu - 2,$$

find an expression for $\left.\frac{\partial u}{\partial x}\right|_{y,z}$.

2. (25) Solve

$$x\frac{dy}{dx} + (xy)^3 = y, \qquad y(1) = 1.$$

3. (25) Find a solution valid at $O(\epsilon)$ to

$$\epsilon\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = x, \qquad y(0) = e^{2\epsilon}.$$

4. (25) Consider

$$x^2 + 2y^2 + z^2 = 1.$$

   (a) Formulate a set of non-linear algebraic equations which can be used to find x and y which maximize z subject to the constraint

$$x + y + \sin(xy) = 1.$$

   (b) Sketch the surface described described by the original equation in (x, y, z) space and the curve describing the constraint in (x, y) space.

   (c) From your sketch, make a rough estimate of x and y which maximize z subject to the above constraint.