AME 561
Examination 2
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1. (25) Given $x \in \mathcal{R}^1, f : \mathcal{R}^1 \to \mathcal{R}^1$,

$$f(x) = \sin x \qquad x \in [0,2],$$

find the first two terms in a Fourier-Legendre expansion of $f(x)$. The first two Legendre polynomials are $P_o(s) = 1, P_1(s) = s, s \in [-1,1]$.

2. (25) Consider $x, u_1, u_2 \in \mathcal{C}^2$. The vectors u_1 and u_2 below form a basis in $\mathcal{C}^2$:

$$u_1 = \begin{pmatrix} i \\ 1 \end{pmatrix}, \qquad u_2 = \begin{pmatrix} i \\ 3i \end{pmatrix}.$$

If

$$x = \begin{pmatrix} 3 \\ 4 \end{pmatrix},$$

express x as a linear combination of the basis vectors.

3. (25) If

$$\frac{d^2y}{dx^2} + y^2 = 1, \qquad y(0) = 0, y(2) = 0,$$

choose an appropriate basis function and find all approximations to solutions for $y(x), x \in [0,2]$ available from a one term collocation method.

4. (25) For $\mathbf{x} \in \mathcal{C}^3, \mathbf{b} \in \mathcal{C}^2, \mathbf{A} : \mathcal{C}^3 \to \mathcal{C}^2$, find the most general value of $\mathbf{x}$ that minimizes $||\mathbf{Ax} - \mathbf{b}||_2$ when

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & i \\ 6 & 3 & 3i \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} i \\ 0 \end{pmatrix}.$$