NAME:
AME 561
Examination 1
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1. (25) If

$$y(x) = \int_0^{x^2} \frac{\exp\left(-s^2\right)}{x^2}\, ds,$$

find $\frac{dy}{dx}$, x which maximizes y, and the value of y at its maximum.

2. (25) Find a solution $y(t)$ for arbitrary $f(t)$ for the following differential equation and initial conditions using the Green's function technique:

$$\frac{d^2y}{dt^2} = f(t); \qquad y(0) = 2, \qquad \dot{y}(0) = 1.$$

Hint: First define a new function, $u(t)$, which is any function that satisfies the initial conditions on $y(t)$, and use the principle of superposition to define a modified problem with homogeneous boundary conditions. Also take as your domain $0 < t < \infty$. Verify your solution if $f(t) = 2t$.

3. (25) Find $y(x)$ which satisfies the following differential equation and initial conditions:

$$\frac{d^3y}{dx^3} + \frac{dy}{dx} = \sin(x); \qquad y(0) = 1, \qquad y'(0) = 0, \qquad y''(0) = 0.$$

4. (25) For $0 < \epsilon << 1$, consider the following differential equation and associated initial condition:

$$\epsilon \frac{dy}{dx} + y = \frac{1}{1 + \epsilon y^2}; \qquad y(0) = 0.$$

Find the composite solution which matches the differential equation and boundary condition at leading order. Find the composite solution which matches the differential equation and boundary condition at the following order.