NAME:
AME 561
Examination 2
Prof. J. M. Powers
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1. (25) For $y(x) \in \mathcal{L}_2[0,1]$, show that the Sturm-Liouville operator $\mathbf{L}_s$, defined such that

$$\mathbf{L}_s y(x) = \frac{1}{r(x)} \left(\frac{d}{dx} \left(p(x) \frac{d}{dx} \right) + q(x) \right) y(x),$$

is self-adjoint in the special case in which $r(x) = p(x) = 1$, along with boundary conditions $y(0) = y(1) = 0$.

2. (25) Let $F(x,y) = x^2 - y^2$. Evaluate

$$\int_{(0,0)}^{(2,8)} \nabla F \cdot d\mathbf{r} \qquad \text{on} \qquad y = x^3.$$

3. (25) For $\mathbf{A} : \mathcal{R}^2 \to \mathcal{R}^3$, find the vector $\mathbf{x} \in \mathcal{R}^2$ of minimum $||\mathbf{x}||_2$ which minimizes $||\mathbf{A}\mathbf{x} - \mathbf{b}||_2$ when

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}.$$

4. (25) For $\mathbf{A} : \mathcal{C}^1 \to \mathcal{C}^2$,

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2i \end{pmatrix},$$

find all terms in the singular value decomposition of $\mathbf{A}$.