AME 561
Examination 1
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1. (25) If
$$u = \frac{x+y}{x-y}, \qquad v = \frac{xy}{(x-y)^2},$$
determine if u is functionally dependent on v.

2. (25) Find all solutions for
$$x\frac{dy}{dx} + 2\left(\frac{dy}{dx}\right)^2 - y = 0, \qquad y(0) = 1.$$

3. (25) Find a solution valid at $O(\epsilon)$ to
$$\frac{d^3y}{dx^3} + \frac{dy}{dx} = \epsilon \sin x,$$
$$y(0) = e^{\epsilon}, \quad y'(0) = 0, \quad y''(0) = 0.$$

4. (25) Find a solution $y(t)$ for arbitrary $f(t)$ for the following differential equation and initial conditions using the Green's function technique:
$$\frac{d^2y}{dt^2} = f(t); \qquad y(0) = 2, \qquad \dot{y}(0) = 1.$$
Take as your domain $0 < t < \infty$. Verify your solution if $f(t) = 2t$.