AME 561
Examination 2
Prof. J. M. Powers
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1. (25) Given $x \in \mathcal{R}^1$, $f : \mathcal{R}^1 \to \mathcal{R}^1$,

$$f(x) = e^x, \qquad x \in [0, 3],$$

find the first two terms in a Fourier-Legendre expansion of $f(x)$. The first two Legendre polynomials are $P_o(s) = 1$, $P_1(s) = s$, $s \in [-1, 1]$.

2. (25) Consider the curve on the unit sphere, $x^2 + y^2 + z^2 = 1$ for which $y = x^2$. Find the curvature κ and torsion τ of the curve at the point $(x, y, z) = (\frac{1}{2}, \frac{1}{4}, \frac{\sqrt{11}}{4})$.

3. (25) For $\mathbf{A} : \mathcal{R}^3 \to \mathcal{R}^2$, find the vector $\mathbf{x} \in \mathcal{R}^3$ of minimum $||\mathbf{x}||_2$ which minimizes $||\mathbf{A} \cdot \mathbf{x} - \mathbf{b}||_2$ when

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

4. (25) Find the Jordan decomposition of $\mathbf{A}$ where

$$\mathbf{A} = \begin{pmatrix} 0 & i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

that is find $\mathbf{P}$ and $\mathbf{J}$ such that $\mathbf{A} = \mathbf{P} \cdot \mathbf{J} \cdot \mathbf{P}^{-1}$, where $\mathbf{J}$ is in Jordan canonical form.