AME 561 Fall 1999 Exam 2

1. $F(x) = e^x$ $x\in[0,3]$, let $y = 2/3\,x-1$, then $x\in[0,3]\to y\in[-1,1]$ & $f(y) = e^{3/2(y+1)}$

Fourier coeff → $A_n = \int_{-1}^{1} F(y)\,\phi_n(y)\,dy$; $\phi_0(y) = \sqrt{\tfrac12}$; $\phi_1(y) = \sqrt{\tfrac32}\,y$ $\quad A_0 = \sqrt{\tfrac12}\int_{-1}^{1} e^{3/2(y+1)}dy$

$$A_0 = \sqrt{\tfrac12}\,\tfrac23\, e^{3/2(y+1)}\Big|_{-1}^{1} = -\frac{\sqrt2}{3} + \frac{\sqrt2}{3}e^3 \;;\; A_1 = \sqrt{\tfrac32}\int_{-1}^{1} y\, e^{3/2(y+1)}dy = \tfrac13\sqrt{\tfrac23}\left(5+e^3\right)$$

$$\therefore\; e^{3/2(y+1)} \sim \boxed{1/3\left(-1+e^3\right) + \tfrac13\left(5+e^3\right)\left(2/3\,x-1\right) \sim e^x}$$

2. Find κ & τ for $x^2+y^2+z^2=1$; $y=x^2$ @ $(1/2, 1/4, \sqrt{11}/4)$. Take $x=t$, $y=t^2$, $z=\sqrt{1-t^2-t^4}$

so we have @ $(1/2, 1/4, \sqrt{11}/4)$, $t=1/2$; $\frac{d\underset{\sim}{r}}{dt} = \underset{\sim}{i} + 2t\underset{\sim}{j} + \tfrac12\left(1-t^2-t^4\right)^{-1/2}\left(-2t-4t^3\right)\underset{\sim}{k}$

@ $t=1/2$ $\quad d\underset{\sim}{r}/dt = \underset{\sim}{i} + \underset{\sim}{j} - 3/\sqrt{11}\,\underset{\sim}{k}$, $\quad d^2\underset{\sim}{r}/dt^2 = 2\underset{\sim}{j} - \dfrac{\left(-2t-4t^3\right)^2}{4\left(1-t^2-t^4\right)^{3/2}} + \dfrac{-2-12t^2}{2\sqrt{1-t^2-t^4}}\,\underset{\sim}{k}$

@ $t=1/2$ $\quad \dfrac{d^2\underset{\sim}{r}}{dt^2} = 0\underset{\sim}{i} + 2\underset{\sim}{j} - \dfrac{146}{11\sqrt{11}}\underset{\sim}{k}$ & $\|\dot r\| = \sqrt{\tfrac{31}{11}}$, $\|\ddot r\| = \tfrac{12}{11}\sqrt{\tfrac{185}{11}}$

$$\kappa = \frac{1}{\|\dot r\|^3}\left(\|\ddot r\|^2\|\dot r\|^2 - (\dot r\cdot\ddot r)^2\right)^{1/2} = \boxed{\tfrac{11}{31}\sqrt{\tfrac{2065}{31}} = 1.05312}$$

$$\tau = -\frac{1}{\kappa^2\|\dot r\|^6}\left((\dot r\times\ddot r)\cdot\dddot r\right) = \boxed{\tfrac{-204\sqrt{11}}{413} = -1.638}$$

3. $\begin{pmatrix}1&1&2\\3&3&6\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$ $\quad \begin{pmatrix}1&3\\1&3\\2&6\end{pmatrix}\begin{pmatrix}1&1&2\\3&3&6\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}1&3\\1&3\\2&6\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix} \Rightarrow \begin{pmatrix}10&10&20\\10&10&20\\20&20&40\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}11\\11\\22\end{pmatrix}$

$\Rightarrow \begin{pmatrix}10&10&20\\0&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}11\\0\\0\end{pmatrix}$ $\quad x_2 = s$, $x_3 = t$ $\quad x_1 = \tfrac{11}{10} - s - 2t$ $\quad \underset{\sim}{x} = \begin{bmatrix}11/10\\0\\0\end{bmatrix} + s\begin{bmatrix}-1\\1\\0\end{bmatrix} + t\begin{bmatrix}-2\\0\\1\end{bmatrix}$

but $\begin{pmatrix}1&-1&-2\\1&1&0\\2&0&1\end{pmatrix}\begin{pmatrix}c_1\\c_2\\c_3\end{pmatrix} = \begin{bmatrix}11/10\\0\\0\end{bmatrix}$ $\quad\therefore\quad c_1 = 11/60$, $c_2 = -11/60$, $c_3 = 11/30$ take only row space component $\quad \boxed{\text{so } \underset{\sim}{x} = \tfrac{11}{60}\begin{bmatrix}1\\1\\2\end{bmatrix}}$

4. $\underset{\sim}{A} = \begin{bmatrix}0&i&0\\i&0&0\\0&0&0\end{bmatrix} \Rightarrow \underset{\sim}{A} - \lambda\underset{\sim}{I} = 0 \Rightarrow \begin{bmatrix}-\lambda&i&0\\i&-\lambda&0\\0&0&-\lambda\end{bmatrix} = 0 \quad -\lambda\begin{vmatrix}-\lambda&i\\i&-\lambda\end{vmatrix} = 0$

$-\lambda(\lambda^2+1) = 0$ $\quad \lambda = 0$, $\lambda = \pm i$ $\quad$ for $\lambda=0$ $\begin{bmatrix}0&i&0\\i&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$e_3 = s$, $e_1 = 0$, $e_2 = 0 \Rightarrow \lambda=0$ $\underset{\sim}{e} = s\begin{bmatrix}0\\0\\1\end{bmatrix}$ $\quad$ for $\lambda = i$ $\begin{bmatrix}-i&i&0\\i&-i&0\\0&0&-i\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$\begin{bmatrix}1&-1&0\\0&0&0\\0&0&-i\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ $\quad \underset{\sim}{e} = t\begin{bmatrix}1\\1\\0\end{bmatrix}$ $\quad$ for $\lambda = -i$ $\begin{bmatrix}i&i&0\\i&i&0\\0&0&i\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$\begin{bmatrix}1&1&0\\0&0&0\\0&0&i\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ $\quad \underset{\sim}{e} = r\begin{bmatrix}1\\-1\\0\end{bmatrix}$ $\quad \boxed{\text{Take } \underset{\approx}{P} = \begin{bmatrix}0&1&1\\0&1&-1\\1&0&0\end{bmatrix} \quad \underset{\approx}{J} = \begin{bmatrix}0&0&0\\0&i&0\\0&0&-i\end{bmatrix}}$

& $\underset{\approx}{P}^{-1} = \begin{bmatrix}0&0&1\\\tfrac12&\tfrac12&0\\\tfrac12&-\tfrac12&0\end{bmatrix}$, $\underset{\approx}{P}\cdot\underset{\approx}{J}\cdot\underset{\approx}{P}^{-1} = \underset{\approx}{A}$