AME 60614
Examination 1
Prof. J. M. Powers
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1. (30) Consider the following data

x	$f(x)$
0	1
1	3
3	6

   Generate a global Lagrange interpolating polynomial to fit the data. Estimate $df/dx|_{x=0}$ with a first order method and then do the same for the highest order estimate that employs the global Lagrange interpolating polynomial.

2. (40) Consider the system

$$\frac{dy_1}{dt} = -2y_1 + y_2, \qquad y_1(0) = 1,$$
$$\frac{dy_2}{dt} = y_1 - 2y_2, \qquad y_2(0) = 0.$$

   Determine if the exact solution is stable. With $\mathbf{y} = (y_1, y_2)^T$, and taking the step size $\Delta t = h$, cast the forward Euler approximation to the solution in the form

$$\mathbf{y}^{(n+1)} = \mathbf{B} \cdot \mathbf{y}^{(n)}.$$

   Find $\mathbf{B}$. Determine a condition on h for the method to be stable.

3. (30) Consider the model problem $dy/dt = \lambda y$, $y(0) = 1$. One can apply the leapfrog method to estimate an approximate soltuion:

$$\frac{y_{n+1} - y_{n-1}}{2h} = \lambda y_n.$$

   Through use of Taylor series, a) find the *modified equation* for which the leapfrog method provides a better approximation, b) show the leapfrog method is a consistent method, c) prepare an exact solution to the modified equation and find the algebraic equation whose solution is required to ascertain the stability of the leapfrog method.