AME 538
Examination 1
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1. (20) Using Cartesian index notation, show the following identity is true:

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$$

2. (40) In a Cartesian coordinate system, after some appropriate non-dimensionalization, a flow has the following velocity components:

$$v_1 = x_1, \qquad v_2 = t, \qquad v_3 = 0.$$

   (a) At $t = 2$, what is the equation of a streakline passing through the point P : $(x_1, x_2) = (2, 2)$?

   (b) (40) At $t = 0$, a fluid particle is located at P. What is the location of that fluid particle at $t = 2$?

   (c) If the fluid is inviscid, subjected to no body force, and is an *isothermal* ideal gas, find an expression for a pressure field which could induce this velocity field.

   (d) For this inviscid isothermal fluid with no body force, what is the time rate of change of the vorticity of the fluid particle which is located at point P at $t = 2$, and what, if any, mechanism is generating the vorticity?

3. (40) Consider a compressible Newtonian fluid that obeys Stokes assumption and Fourier's law. Further assume that the fluid is a calorically perfect ideal gas. Starting with the *conservative form* of the energy conservation equation and employing all other necessary conservation laws, derive an expression for the material derivative of temperature, $\frac{dT}{dt}$. Use Cartesian index notation, and show all steps in your analysis.