AME 538
Examination 1
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1. (20) Using Cartesian index notation, show the following identity is true:

$$(\mathbf{v} \cdot \nabla)\mathbf{v} = \nabla \left(\frac{\mathbf{v} \cdot \mathbf{v}}{2} \right) - \mathbf{v} \times (\nabla \times \mathbf{v}).$$

2. (30) In a Cartesian coordinate system, a flow has the following velocity components:

$$v_1 = \alpha x_2, \qquad v_2 = \beta x_1^2, \qquad v_3 = 0.$$

   Here α and β are constants with appropriate units.

   (a) Find the equation of a streamline passing through the point $P : (x_1, x_2) = (1, 1)$.

   (b) Find the principal values of strain rate at P.

   (c) If the fluid is Newtonian, but does not satisfy Stokes' assumption, find the viscous stress tensor, then identify the mean viscous stress and the deviatoric viscous stress.

3. (30) Starting with the non-conservative form of the energy equation

$$\rho \frac{de}{dt} = -\partial_i q_i - p\partial_i v_i + \tau_{ij}\partial_i v_j,$$

   (a) Show all steps in an analysis which finds the mechanisms which induce the entropy of a fluid to change.

   (b) Using words only, compare the mechanisms which induce entropy changes to those which induce fluid rotation.

4. (20) For in incompressible Newtonian fluid with constant properties, show that

$$\left(\nabla^T \cdot \boldsymbol{\tau}\right)^T = -\mu \left(\nabla \times \boldsymbol{\omega}\right).$$