

1. (20) A uniform stream with velocity U in the x direction combines with a source of strength m at $(a, 0)$ and a sink of strength $-m$ at $(-a, 0)$. Plot the resulting streamlines, note any stagnation points and closed-body streamlines. If the far field pressure is p_o , find the pressure at the leading edge stagnation point.
2. (20) A complex potential is given by

$$W(z) = 2z + \frac{1}{z} + \frac{1}{z^2}.$$

Find the velocity vector at the point $(x, y) = (1, 1)$.

3. (30) A shock is driven into air by a piston moving at 300 m/s . The still air has a temperature of 300 K and a pressure of 100 kPa .
 - (a) Assuming air to be a calorically perfect ideal gas with $R = 287 \text{ J/kg/K}$ and $\gamma = 7/5$, calculate the speed of the shock, and the pressure after the passage of the shock.
 - (b) Assuming air to be an ideal gas with $R = 287 \text{ J/kg/K}$ and calorically imperfect with

$$e = a_0 + a_1T + a_2T^2,$$

with $a_0 = 4640 \text{ J/kg}$, $a_1 = 706 \text{ J/kg/K}$, $a_2 = 0.062 \text{ J/kg/K}^2$, pose the resulting system of non-linear algebraic equations which could be used to solve for the shock speed and post-shock pressure, as well as other variables. Do not solve.

4. (30) Sir Isaac Newton, living in an era in which Boyle's Law and Charles' Law were well understood, but entropy and basic thermodynamic principles were not, was mistakenly inclined to think of gas dynamics as an isothermal process. Assuming the energy equation is replaced in favor of an isothermal condition,
 - (a) Write the mass and momentum equations for one-dimensional inviscid, unsteady, flow of a calorically perfect ideal gas, reducing them to be in terms of the two unknowns ρ and u .
 - (b) Write these equations in characteristic form.
 - (c) Use this flawed theory to estimate the speed of sound.
 - (d) Compare a Newtonian estimate of the speed of sound in calorically perfect ideal air at 300 K to that of an isentropic theory.