NAME:
AME 538
Examination 1
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## Index Notation (15)

1. Using Cartesian index notation, show the following identity is true:

$$
\nabla \times(\nabla \times \mathbf{a})=\nabla(\nabla \cdot \mathbf{a})-\nabla^{2} \mathbf{a}
$$

Fluid Kinematics/Stress Tensor (70)
2. In a Cartesian coordinate system, a steady flow has the following velocity components:

$$
v_{1}=k\left(x_{1}^{2}+x_{2}^{2}\right), \quad v_{2}=k\left(x_{1}^{2}-x_{2}^{2}\right), \quad v_{3}=0
$$

The fluid is known to be Newtonian, satisfies Stokes assumption, and has a first coefficient of viscosity $\mu$. Take $k=100 \mathrm{~m}^{-1} \mathrm{~s}^{-1}, \mu=0.001 \mathrm{Ns} / \mathrm{m}^{2}$. Take the thermodynamic stress to be zero. Consider the point $P:(10 m, 5 m, 0 m)$.
a) What is the solid body rotation rate of a fluid element at $P$ ? (5)
b) Decompose the relative velocity of a neighboring point $P^{\prime}:(10 m, 6 m, 0 m)$ into velocity vectors attributible to rotation, extensional strain, and shear strain. Sketch a vector diagram showing all components. (15)
c) What is the viscous stress tensor, $\tau_{i j}$ at $P$ ? (10)
d What is the viscous stress vector, $R_{j}$ associated with the plane whose unit normal lies in the direction of $P^{\prime}$ with respect to $P$ ? (10)
3. Given the stress tensor $T_{i j}$ below,
a) find the principal values and the orientation of the axes on which the principal values exist. (15)
b) find the magnitude of the shear stress for an orientation in which the normal stress is zero. (15)

$$
T_{i j}=\left[\begin{array}{ll}
1 & 1 \\
1 & 2
\end{array}\right]
$$

## Governing Equations (15)

4. State in words the axiom of mass conservation and using the method developed in class, derive the partial differential equation which embodies this principle: (15)

$$
\partial_{o} \rho+\partial_{i}\left(\rho v_{i}\right)=0 .
$$

