NAME:
AME 538/ME 438
Examination 1
Prof. J. M. Powers
October 4, 1994

Index Notation (20)

1. Using Cartesian index notation, show the following identity is true:

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

2. Kinematics (20)

2. In a Cartesian coordinate system, a steady flow has the following velocity components:

$$v_1 = x_2, \qquad v_2 = x_1, \qquad v_3 = 0$$

a) What is the equation of a streamline passing through the point $P : (x_1, x_2) = (2, 2)$?
b) Sketch the streamline through P, and all the streamlines in the x_1, x_2 plane.
c) At $t = 0$, a fluid particle is located at P. What is the location of that fluid particle at $t = 2$?

3. Stress Tensor (20)

The symmetric total stress tensor T_{ij} in a two-dimensional problem is

$$T_{ij} = \begin{bmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{bmatrix}$$

If the component of stress normal to an *arbitrary* plane with unit normal $n_i = [n_1, n_2]^T$ is to be a *constant* value, $\hat{T}$, clearly demonstrate what each independent component of the stress tensor, T_{11}, T_{12}, T_{22}, must be in order for this to hold.

4. Vorticity Transport (20)

4. Starting with the linear momentum principle for an incompressible ($\rho = \rho_o$) fluid with no viscous stress ($\tau_{ij} = 0$), subjected a constant body force g_i :

$$\rho_o \frac{dv_i}{dt} = -\partial_i P + \rho_o g_i$$

derive the appropriate Helmholtz vorticity transport equation and give a *brief* description of the mechanisms which cause vorticity to change.

5. Entropy Generation (20)

For an incompressible constant viscosity Newtonian fluid that satisfies Stokes' assumption, the velocity field is given by $v_1 = v_o[1 - (x_2/h)^4], v_2 = 0, v_3 = 0$. Take $v_o = 2.0\ m/s$, $h = 1\ m$, $\rho = 1.0\ kg/m^3$ and $\mu = 0.001\ Ns/m^2$. The fluid is assumed to be isothermal, held at $T = 300\ K$. At $x_2 = 0.5\ m$, what is the local rate of entropy generation, ds/dt, due to viscous forces?