AME 538, ME 438
Test 2
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1. (25) Consider the centered Prandtl-Meyer rarefaction induced by a piston suddenly accelerated from $0\ m/s$ to $-500\ m/s$. The ambient fluid is air at $T_o = 300\ K$, $P_o = 10^5\ Pa$, $u_o = 0\ m/s$. Take $R = 287\ J/(kgK), \gamma = 7/5$.

Sketch the x, t plane with the relevant characteristics.

At $(x, t) = (50m, 2s)$ what is the instantaneous local fluid velocity and pressure?

2. (25) Starting with the linear momentum principle for an incompressible Newtonian fluid under the influence of a constant body force, derive the appropriate Helmholtz vorticity transport equation.

3. At $t = 0$, two ideal point vortices, each of strength Γ, are positioned on the x axis, straddling the origin. One vortex is located at $(a, 0)$ and has a counterclockwise orientation, the other is at $(-a, 0)$ and has a clockwise orientation. Take the body force to be zero, the constant fluid density to be ρ, and the viscosity to be negligible.

a. In order that the vortices remain on the x axis for all time, what must be the magnitude and direction of the freestream velocity?

b. What is the velocity potential $\phi(x, y)$ induced by the combination of the freestream and two vortices?

c. What is the pressure field $P(x, y)$?

4. An incompressible liquid flows with constant velocity $v_i(x_i, t) = (U, 0, 0)$. At $x = -\infty$ the fluid has temperature T_∞. The fluid passes through a very thin, very fine-mesh screen located at $x = 0$ whose unit normal is $n_i = (1, 0, 0)$. The screen is held at constant temperature T_s, and you can assume that the fluid temperature at $x = 0$ is in fact T_s. Neglecting viscous dissipation, and taking the liquid's density, specific heat, and thermal conductivity to be ρ, c, and k, respectively, the energy equation becomes

$$\rho c U \frac{dT}{dx} = k \frac{d^2 T}{dx^2}$$

Write this equation in dimensionless form defining appropriate dimensionless variables and parameters. Also define appropriate dimensionless boundary conditions.