AME 538, ME 438
Test 2
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1. (20) Glycerin, $\rho = 1260 \quad kg/m^3$, $\mu = 1.49 \ (Ns)/m^3$, is flowing under the influence of gravity, $g = 9.81 \ m/s^2$, down a very long plane inclined at an angle $\theta = \pi/6$ to the horizontal at the rate of $\dot{m} = 10 \ kg/s$ (see Fig. 1). Assume the flow is fully developed, there is no pressure gradient, and that the shear force at the interface between the glycerin and the atmosphere is zero. In this problem, leave all variables and results dimensional.

a) Write the second order ordinary differential equation and two boundary conditions to determine $u(y)$.

b) Express the velocity profile $u(y)$ in terms of ρ, μ, θ, and the as of yet unknown film thickness h.

c) What must the numerical value of film thickness h be in order that mass be conserved.?

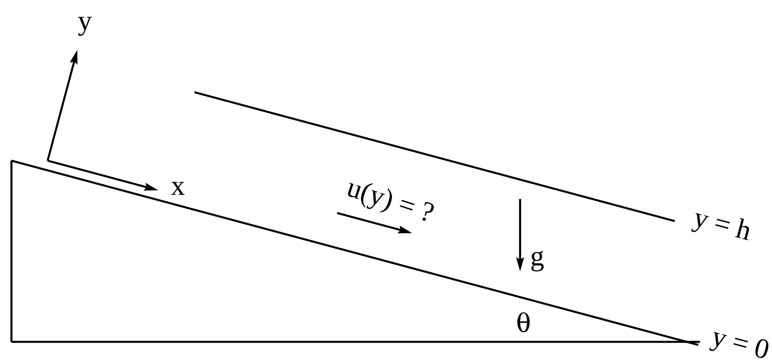

Figure 1: Schematic for flow down inclined plane.

2. (20) A fluid with constant density ρ, specific heat c, viscosity μ, and thermal conductivity k is in a semi-infinite domain bounded at $y = 0$ by a flat plate (see Fig. 2. At $t = 0$ both the plate and fluid are at rest and at temperature T_o. At $t = 0^+$, the plate is suddenly heated to T_1. A similarity solution exists; the similarity variable is $\eta = y/\sqrt{\frac{k}{\rho c}t}$. What is $u(y,t)$ and $T(y,t)$ in the fluid?

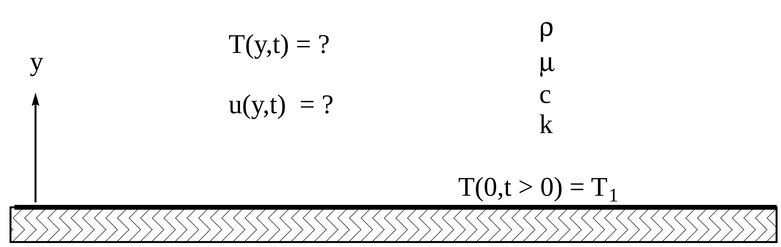

Figure 2: Schematic for fluid heating problem.

3. (20) If the complex velocity potential is the superposition of a source and uniform freestream as follows,

$$W(z) = U_o z + \frac{m}{2\pi} \ln z,$$

the constant density is ρ, and the pressure in the far-field is P_o, what is the pressure at the point $(x, y) = (1, 2)$?

4. (20) Sketch the streamlines of a uniform stream at U_o past a sink of strength $-2m$ at the origin, a source of strength m at $(a, 0)$, and a source of strength m at $(-a, 0)$. Does a closed body shape form? You do not need to do or show any calculations for this problem.

5. (20) A fluid of constant density ρ and viscosity μ is initially at rest in the annular region between two concentric cylinders (see Fig. 3). At $t = 0$, the inner cylinder begins to rotate with constant angular velocity Ω, while the outer cylinder remains at rest. The inner and outer radii are R_i and R_o, respectively.

a) Assuming that the only non-zero velocity component is $v_\theta(r, t)$, and that the pressure field is constant, write the dimensional θ momentum equation, initial and boundary conditions.

b) Define appropriate scales, and write the partial differential equation, initial, and boundary conditions in dimensionless form. Do not solve the equation.

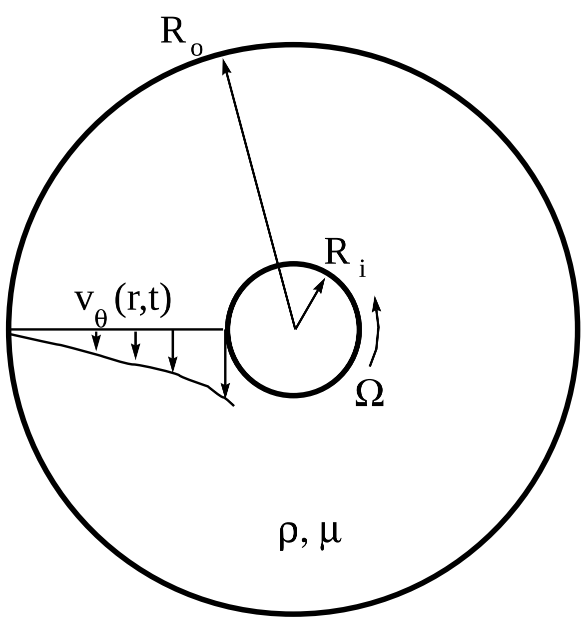

Figure 3: Schematic of rotating cylinder problem.