AME 60636
Prof. J. M. Powers
Homework 9
Due: Monday, 6 November 2006

1. Consider a slab of the solid energetic material LX-14 (a common explosive). The slab has length $L = 0.25\ m$. Assume the LX-14 has material properties as given by Powers [1], with the following exceptions, which we take to avoid problems of numerical convergence, $a = 5 \times 10^{-5}\ s^{-1}, E = 2.206 \times 10^{4}\ J/mol$. Consider the Frank-Kamenetskii problem for this scenario. Assume the temperature at the outer radius is held fixed at 300 K and the temperature evolution is governed by the following differential equation as developed in lecture:

$$\frac{\partial T}{\partial t} = \frac{1}{\mathcal{D}} \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) + (1 - T) \exp \left(\frac{-\Theta}{1 + QT} \right).$$

   (a) Use a numerical shooting technique to solve for the temperature distribution $T(x)$ in the limit of steady state.
   (b) Holding other parameters fixed, vary $\mathcal{D}$ and plot $T(x = 0)$ as a function of D.
   (c) Find the critical slab length below which small temperature solutions may exist.
   (d) Use the technique developed in class to consider the linear stability of all three steady solutions to the Frank-Kamenetskii problem. Plot a few eignemodes for each solution and label the plots with the associated value of the eigenvalue. From the eigenvalues, estimate the time constant of the most unstable mode of each steady solution.

[1] Powers, J. M., 1999, "Thermal explosion theory for shear localizing energetic solids," *Combustion Theory and Modelling*, Vol. 3, pp. 103-122.