AME 60636
Prof. J. M. Powers
Homework 7
Due: Friday, 23 March 2012

1. Consider a mixture of CH_4 and O_2. Initially, the mixture is at 298.15 K and 100 kPa. The mixture is in a fixed, closed, adiabatic vessel with $V = 1\ m^3$. Assuming the only possible products of combustion are CO_2, H_2O, O_2 and CH_4, give a plot of adiabatic flame temperature as a function of equivalence ratio.

2. Consider a slab of the solid energetic material LX-14 (a common explosive). The slab has $L = 0.25\ m$, and has total length $2L$. Assume the LX-14 has material properties as given by Powers[1], with the following exceptions, which we take to avoid problems of numerical convergence, $a = 5 \times 10^{-5}\ s^{-1}$, $\overline{E} = 2.206 \times 10^4\ J/mol$. Solve the Frank-Kamenetskii problem for this scenario. Assume the temperature at the outer boundaries is held fixed at 300 K, the initial temperature distribution is spatially uniform at 300 K, and the temperature evolution is governed by the following dimensionless differential equation as developed in lecture:

$$\frac{\partial T}{\partial t} = \frac{1}{\mathfrak{D}} \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + (1 - T) \exp\left(\frac{-\Theta}{1 + QT}\right).$$

   (a) Use a numerical shooting technique to solve for the temperature distribution $T(x)$ in the limit of steady state.

   (b) Holding other parameters fixed, vary $\mathfrak{D}$ and plot $T(x = 0)$ as a function of $\mathfrak{D}$.

   (c) Find the critical slab length below which small temperature solutions may exist.

[1] Powers, J. M., 1999, "Thermal explosion theory for shear localizing energetic solids," *Combustion Theory and Modelling*, Vol. 3, pp. 103-122.