AME 60636
Prof. J. M. Powers
Homework 8
Due: Friday, 30 March 2012

1. Consider a slab of the solid energetic material LX-14 (a common explosive). The slab has length $L = 0.25\ m$. Assume the LX-14 has material properties as given by Powers [1], with the following exceptions, which we take to avoid problems of numerical convergence, $a = 5 \times 10^{-5}\ s^{-1}, \overline{E} = 2.206 \times 10^{4}\ J/mol$. Consider the Frank-Kamenetskii problem for this scenario. Assume the temperature at the outer radius is held fixed at 300 K and the temperature evolution is governed by the following differential equation as developed in lecture:

$$\frac{\partial T}{\partial t} = \frac{1}{\mathcal{D}} \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) + (1 - T) \exp \left(\frac{-\Theta}{1 + QT} \right).$$

   (a) Use the technique developed in class to consider the linear stability of all three steady solutions to the Frank-Kamenetskii problem. Plot a few eigenmodes for each solution, and label the plots with the associated value of the eigenvalue. From the eigenvalues, estimate the time constant of the most unstable mode of each steady solution.

   (b) Use an appropriate numerical technique such as the method of lines to solve the full unsteady initial value problem if $T(x, 0) = 300\ K$. Comment on whether the time scales of relaxation compare well with those determined from your linear stability analysis.

2. Taking $T_{IG} = 0.1$, and otherwise using the same model, approach, and parameters as given in the course notes for a premixed burner-stabilized one-dimensional steady laminar flame, generate plots of $T(x)$, $\rho(x)$, $Y_B(x)$, and $u(x)$.

[1] Powers, J. M., 1999, "Thermal explosion theory for shear localizing energetic solids," *Combustion Theory and Modelling*, Vol. 3, pp. 103-122.