Physical Diffusion Suppresses the Carbuncle Instability

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APS Sixty-Eighth Annual DFD Meeting

Boston, Massachusetts

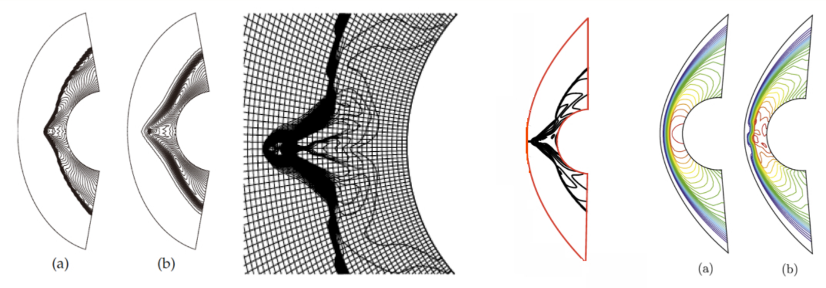
24 November 2015

Support from the University of Notre Dame FlowPAC

Carbuncle Phenomenon I

Anomalous solutions in shock-capturing methods in Euler equations:

- High amplitude incongruity in the neighbourhood of the shock's axis of symmetry



$M_\infty = 20$
Chandrashekar
CCP, 2013

$M_\infty = 10$
Dumbser
JCP, 2004

$M_\infty = 10$
Li et al.
JCP, 2011

$M_\infty = 6$
Srinivasan et al.
IJNMF, 2012

Carbuncle Phenomenon II

- Not observed in nature, a spurious solution of numerical origin
  - Either, an anomaly of the chosen numerical method
  - Or, an inadequacy of the underlying mathematical model
- "Unconditional instability with exponential error growth"
  - Dumbser, et al. *JCP*, 2004
  - Robust matrix stability analysis
  - Carbuncle phenomenon is sensitive to M_∞, high order scheme, Euler equation, time advancement scheme, CFL number
- "Incurable"
  - Elling, *Acta Mathematica Scientia*, 2009

Remedies

- Physical diffusion
  - “Even for unpractically low *Re*”, still...
    - Underresolved viscous shock
    - Pandolfi and D’Ambrosio, *JCP*, 2001
  - “Disappears only at very low Reynolds number”
    - Viscous cure
    - Discounted it
    - Ismail et al. *ICCFD4*, 2009
- Kinetic theory
  - Ohwada et al. *JCP*, 2013, Li et al. *JCP*, 2011
- Hybrid method, intricate numerical algorithm in NS equations
  - Chandrashekar, *CCP*, 2013, Nishikawa and Kitamura, *JCP*, 2008
- Spectral Solution
  - Kopriva, *AIAA J*, 1993, Hejranfar et al. *JCP*, 2009

Motivation

We are investigating the supersonic flow of a calorically perfect ideal gas past a two-dimensional blunt body:

- This study will predict the supersonic flow around a cylinder by improving upon prior methods of computation.
- Strategies for computing such flows with Euler equations lead to the “carbuncle phenomenon”.
- We find a simple viscous antidote that will avoid numerical anomalies through a damping mechanism to suppress instabilities.
- This physically based approach can also insure fidelity with what is observed in nature.

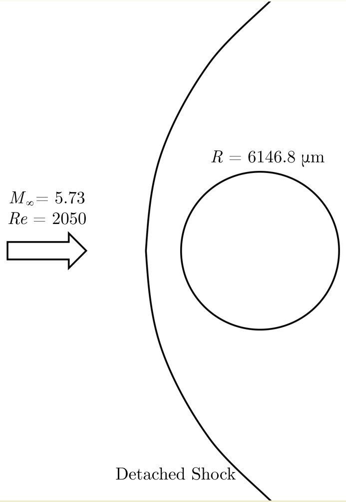
Governing Equations

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0, \\
\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot \left(\rho \mathbf{u}\mathbf{u}^T\right) &= -\nabla p + \nabla \cdot \boldsymbol{\tau}, \\
\frac{\partial}{\partial t}\left(e + \frac{1}{2}\mathbf{u} \cdot \mathbf{u}\right) + \nabla \cdot \left(\rho \mathbf{u}\left(e + \frac{1}{2}\mathbf{u} \cdot \mathbf{u}\right)\right) &= -\nabla \cdot \mathbf{q} - \nabla \cdot (p\mathbf{u}) + \nabla \cdot (\boldsymbol{\tau} \cdot \mathbf{u}), \\
\boldsymbol{\tau} &= 2\mu \left(\frac{\nabla \mathbf{u} + (\nabla \mathbf{u})^T}{2} - \frac{1}{3}(\nabla \cdot \mathbf{u})\,\mathbf{I}\right), \\
p &= \rho R T, \\
e &= c_v T.
\end{aligned}
$$

Numerical Method

- The same grid was used for viscous and inviscid simulations.
- Mesh refinement study was done to examine grid convergence.
- For state variables of the finite-volume cells to the "left" and "right" of the cell face, constant interpolation is used and will result in a spatially 1^{st} order flux. No limiters are used.
- For flux at cell faces:
  - Inviscid flux: Roe flux splitting without entropy fix is used.
  - Viscous flux: Central differencing is used.
- 4^{th} order explicit Runge-Kutta method is used for time marching.
- Global time step is used in the whole computational domain to meet the requirement of dissipation time scale.
  - Dissipation time scale (finest grid): $\Delta t = \Delta x^2/\nu \approx 4.5 \times 10^{-10}$s.

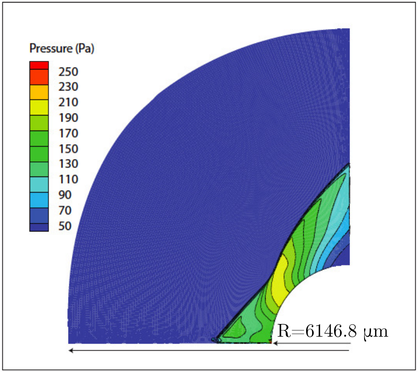
Physical Parameters

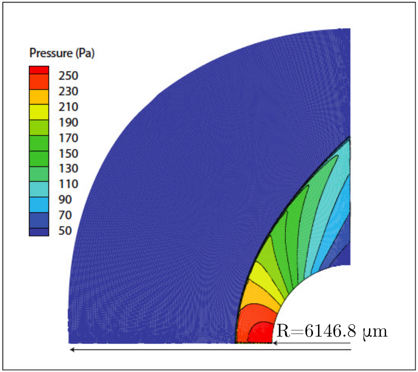


parameter	value	units
M_∞	5.73	
Re	2050	
Pr	0.77	
γ	7/5	
R_s	287.058	J/kg/K
c_v	717.645	J/kg/K
c_p	1004.703	J/kg/K
p_∞	12.7408	Pa
T_∞	39.6667	K
ρ_∞	0.001119	kg/m^3
U_∞	723.4630	m/s
c_∞	126.2588	m/s
μ	2.4272×10^{-6}	Pa s
k	0.003167	W/m/K
α	0.002817	m^2/s

* Parameters were the same as Kopriva, *AIAA J*, 1993, and Tewkit and Giedt, *JAS*, 1960.

Carbuncle versus Non-carbuncle





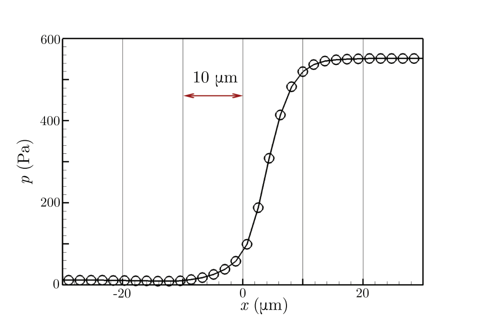
Inviscid

Viscous

Viscous Shock Structure

Moretti, Salas, The blunt body problem for a viscous rarefied gas flow, AIAA Paper 69-139, 1969.

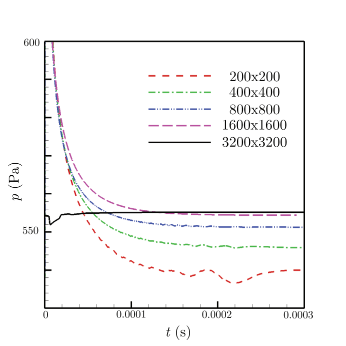
$$\begin{aligned} \delta_s &= \text{shock thickness}/R \\ &\approx \frac{8}{3Re}\frac{\gamma}{\gamma+1}\frac{M_\infty^2-1}{1+\gamma M_\infty^2 - M_\infty\sqrt{2(\gamma+1)(1+M_\infty^2(\gamma-1)/2)}} \\ \text{shock thickness} &\approx 11.9413\ \mu\text{m} \end{aligned}$$



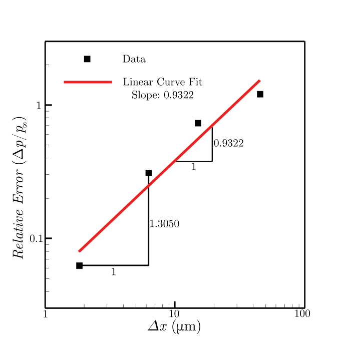
pressure distribution along centerline in front of the circular cylinder

Grid Convergence

Grid #	Dimension	Cell Size (μm)	mpi-n	minutes per $3 \times 10^{-4} s$
1	200×200	45.4160	16	623.7
2	400×400	15.1388	16	1238.1
3	800×800	6.2920	32	11480.0
4	1600×1600	1.8444	64	24091.0
5	3200×3200	0.9922	160	40265.0*
		shock thickness : 11.9413 μm		*27 days 23 hours 5 minutes



Solutions are time-relaxed.
(pressure at cylinder nose)



Solutions in the asymptotic convergence regime in space.

Shock Stand-off Distance

- For $M_\infty = 5.73$, the shock stand-off distance prediction is verified mathematically, but it is not validated against experiment.
- Ambrosio and Wortman, "Stagnation point shock detachment distance for flow around spheres and cylinders". *ARSJ.* 32:281, 1962.

$$\begin{aligned} \text{Wedge} - \text{cylinders}: \qquad \Delta/R &= 0.386\exp(4.67/M_\infty^2) \\ &= 0.444999318 \\ \Delta &= 2735.322\ \mu\text{m} \end{aligned}$$

- $\Delta_{CFD} = 1599.36\ \mu\text{m}$
- Simulation is converged. Shock distance between the experiment and this simulation is 1136 μm, which is 41.53% off.
- This is likely due to neglected physics such as temperature-dependent properties (specific heat, especially).

Conclusion

- A simple physical diffusion model on a sufficiently fine mesh can remove the carbuncle phenomenon;
- Viscous shock structure can be predicted well using a sufficiently fine mesh with Navier-Stokes equations;
- Carbuncle may arise due to what amount to “anti-diffusion” effect;
- Solution is verified;
- Model is not validated yet:
  - Additional physical effects likely need to be included for validation at M_∞=5.73.