Methodology and Analysis for Determination of Propagation Speed of High-Speed Propulsion Devices *

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Abstract

This study describes a methodology and gives an example of a simplified analysis to determine the steady propagation speed of a projectile fired into a gaseous mixture of fuel and oxidizer. For tractability, the steady supersonic flow of an inviscid calorically perfect ideal reacting gas with high activation energy over a symmetric double wedge, unconfined by a cowl, is considered. A search of parameter space reveals propagation speeds which give rise to shocks of such strength which induce a flame sheet to be at a location which allows the combustion-induced thrust to balance the wave drag. For a fixed heat release greater than a critical value, two steady propagation speeds are predicted. The solution at the higher Mach number is stable to quasi-static perturbations while the solution at the lower Mach number is unstable. This methodology is that which should be applied to analyze devices which have more complex geometries such as the ram accelerator or oblique detonation wave engine.

Introduction

It is possible to employ oblique shock waves to induce combustion to generate thrust. Recent discussion has been motivated by the ram accelerator, which has been used to propel projectiles to high speeds, and the oblique detonation wave engine (ODWE), which has been proposed to propel the National Aerospace Plane (NASP). For such devices, it is of fundamental importance to have a theory which can predict a steady propagation speed.

Recent theoretical studies[1−5] related to ram accelerators and ODWE's have not given analysis to determine a steady propagation speed. Typically these studies treat the related problem of flow with a fixed incoming Mach number over a fixed geometry and concentrate on discussing the features of the resulting flow field. Only a small number of incoming Mach numbers are studied. The problems posed are physical in the sense that one could envision an experiment in which the projectile is fixed in a wind tunnel in which the incoming Mach number is controllable. Such an approach, however, says nothing about what the steady speed of a freely propagating vehicle should be. Additionally, of the cited studies, only that of Yungster[2] considers a tip-to-tail projectile geometry, which is necessary to determine the net force. The experimental studies of Hertzberg, et al.[6,7] report observations of projectile velocities in ram accelerators up to $2,500$ m/s in a 16 m tube but also show that the projectile is continuing to accelerate when the end of the tube is reached.

In this paper we describe a general methodology for determining the steady propagation speed of either ram accelerator projectiles or ODWE-powered aerospace planes, describe a simple model problem to illustrate the technique, describe standard jump conditions used to evaluate surface forces, develop an estimate of the induction zone length based upon thermal explosion theory, give our results, and recommend an approach for actual physical scenarios.

Methodology and Model Problem

The chosen approach is to first select a model for the conservation principles and constitutive behavior. Next one chooses a projectile geometry and fluid properties (heat release, kinetic parameters, specific heats, etc.) The model equations are studied in the reference frame in which the projectile is stationary; thus, the incoming flow velocity, which is the steady propagation speed, is thought of as an adjustable parameter at this stage. For a given incoming velocity, solution of the model equations leads to a pressure distribution on the projectile surface which may or may not result in a net force on the projectile. Should the particular incoming velocity lead to zero net force on the projectile, that velocity is a candidate for a steady propagation speed. The quasi-static stability of the candidate solutions is easily determined.

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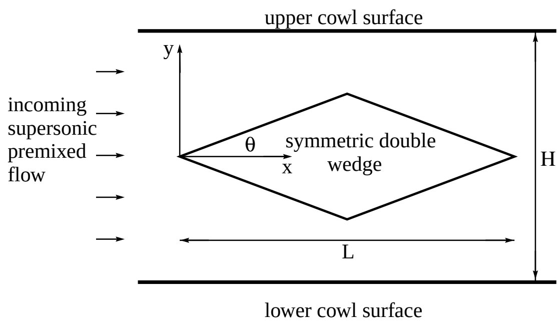

Figure 1: Schematic of generic configuration

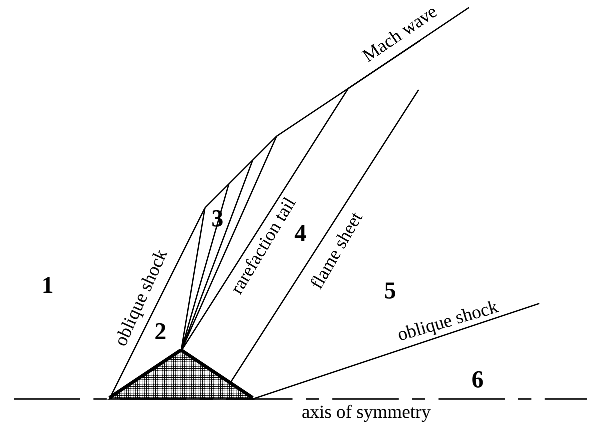

Figure 2: Detailed schematic for $H \to \infty$

Should a perturbation in the incoming velocity lead to a net force which tends to restore the projectile to its speed at which there is zero net force, the solution is stable in a quasi-static sense; otherwise the solution is unstable.

We illustrate this methodology through the use of a model problem which is related to the ram accelerator and ODWE. For tractability, we consider a highly simplified model and geometry which retains the essential physics of the real devices. The geometry, shown in Figure 1, is a symmetric double wedge with half angle θ and length L. Two cowl surfaces are placed symmetrically about the wedge and are separated by height H. The depth of the double wedge and cowl is taken to be infinite and the flow is assumed to have no variation in this direction. The Cartesian coordinate system, with its origin at the leading edge and with the x axis aligned with the incoming flow is also indicated. It is appropriate to think of a ram accelerator as the axisymmetric analog of Figure 1 in which the projectile moves while the cowl is stationary; likewise, an aerospace plane powered by an ODWE can be thought of as the axisymmetric analog of Figure 1 in which the cowl moves with the wedge. In both scenarios one must assume that the incoming fuel and oxidizer are completely mixed; in actuality this is more appropriate for the ram accelerator than the ODWE.

Analysis of the geometry of Figure 1 leads in general to a complicated interaction of shocks, rarefactions, and combustion processes as the flow propagates between the projectile and cowl surface. To further simplify, we only consider the limit $H \to \infty$, Figure 2. Consequently, our geometry shares only the most rudimentary resemblence to actual devices, but has the advantage of being amenable to simple analysis.

Again for tractability, the flow model employed also has only a rudimentary resemblence to commonly used models for real devices. We consider a calorically perfect ideal reacting gas with one-step irreversible Arrhenius kinetics in the high activation energy limit. As a result, we are able to break the flow into discrete regions as shown in Figure 2. The ambient fluid in Region 1 encounters an attached oblique shock at the leading edge. No appreciable reaction occurs within the shock or in Region 2, near the front face. The flow is turned through a centered Prandtl-Meyer expansion in Region 3 till it attains a velocity parallel to the lee wedge surface in Region 4. We take the reaction to occur in a flame sheet which is perpendicular to the lee surface. The location of this flame sheet is given by an induction zone length based upon thermal explosion theory and is related to the incoming Mach number and kinetic parameters. A Rankine-Hugoniot deflagration analysis gives the flow variables in Region 5 based upon values in Region 4 and the heat released in the flame sheet. The flow passes through a final oblique shock to Region 6 where it attains a velocity with only an x component. The net force is then determined by integrating the pressure over the entire surface area.

Model Equations

The model equations are taken to be the unsteady Euler equations and species evolution equation for a reactive calorically perfect ideal gas. These are expressed in dimensionless form using Cartesian index notation:

$$\frac{d\rho}{dt} + +\rho \frac{\partial v_i}{\partial x_i} = 0, \tag{1}$$

$$\rho \frac{dv_i}{dt} + \frac{\partial P}{\partial x_i} = 0, \tag{2}$$

$$\frac{dP}{dt} - \gamma \frac{P}{\rho} \frac{d\rho}{dt} = \frac{(\gamma - 1)\rho\kappa q(1-\lambda)}{M_0^2} \exp\left(\frac{-\Theta}{M_0^2 T}\right) \tag{3}$$

$$\frac{d\lambda}{dt} = \kappa\,(1-\lambda)\exp\left(\frac{-\Theta}{M_0^2 T}\right), \tag{4}$$

$$e = \frac{1}{\gamma-1}\frac{P}{\rho} - \frac{\lambda q}{M_0^2}, \tag{5}$$

$$P = \rho T. \tag{6}$$

The variables contained in Eqs. (1–7) are the density ρ, the Cartesian velocity component v_i, the pressure P, the temperature T, the internal energy e, the reaction progress variable λ, and the Cartesian position coordinate x_i. Here the substantial derivative $\frac{d}{dt} = \frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x_i}$ The freestream Mach number is M_0. Other dimensionless parameters include the ratio of specific heats γ, a kinetic parameter κ, the heat of reaction q, and the activation energy Θ. Equations (1–3) represent the conservation of mass, momenta, and energy, respectively. Equation (4) is a species evolution equation which incorporates an Arrhenius depletion model. Equations (5–6) are caloric and thermal equations of state. A single, first-order, irreversible, exothermic reaction is employed, $A \to B$. The reaction progress variable λ ranges from zero before reaction to unity at complete reaction. Species mass fractions, Y_i are related to the reaction progress variable by the formulae, $Y_A = 1-\lambda$, $Y_B = \lambda$. Initial pre-shock conditions are specified as $\rho = 1$, $u = \sqrt{\gamma}$, $v = 0$, $P = 1/M_0^2$, and $\lambda = 0$.

Equations (1-6) have been scaled such that in the hypersonic limit ($M_0^2 \to \infty$) the pressure, density, and velocities are all $O(1)$ quantities behind the lead shock. The geometric length of the projectile (L) is chosen as the reference length scale. In terms of dimensional variables (indicated by the notation "$\tilde{}$") and dimensional pre-shock ambient conditions (indicated by the subscript "0"), the dimensionless variables are defined by

$$\rho = \frac{\tilde{\rho}}{\tilde{\rho}_0}, \qquad P = \frac{\tilde{P}}{M_0^2 \tilde{P}_0},$$

$$u = \frac{\tilde{u}}{M_0\sqrt{\tilde{P}_0/\tilde{\rho}_0}}, \qquad v = \frac{\tilde{v}}{M_0\sqrt{\tilde{P}_0/\tilde{\rho}_0}},$$

$$x = \frac{\tilde{x}}{L}, \qquad y = \frac{\tilde{y}}{L}. \tag{7}$$

Remaining dimensionless parameters are defined by the following relations:

$$q = \frac{\tilde{\rho}_0 \tilde{q}_1}{\tilde{P}_0}, \qquad \Theta = \frac{\tilde{\rho}_0 \tilde{E}}{\tilde{P}_0}, \qquad \kappa = \frac{\tilde{k}}{\frac{M_0}{L}\sqrt{\frac{P_0}{\rho_0}}}, \tag{8}$$

Here, $\tilde{E}$ is the dimensional activation energy, $\tilde{q}$ is the dimensional heat of reaction, and $\tilde{k}$ is the dimensional kinetic rate constant.

Jump Relations

A series of jump relations can be developed from Eqs. (1-6) to determine the pressure on each surface as a function of the incoming Mach number. We take the high activation energy limit, $\Theta >> 1$, so that it is proper to describe the entire combustion process as a thin flame sheet and choose kinetic parameters, Θ, κ, q such that estimates from thermal explosion theory place the flame sheet on the lee wedge surface. Consequently, it is possible to use standard relations[8] for inert oblique shocks and centered Prandtl-Meyer expansions to determine the pressure in Regions 2, 3, and 4. For the oblique shock between Regions 1 and 2, the weak solution branch is chosen so as to match to a Mach wave at distances far from the projectile. The flow expands in a Prandtl-Meyer expansion from Region 2 through Region 3 until the flow velocity is parallel to the lee wedge surface in Region 4. With the assumption that the flame sheet is perpendicular to the wedge surface, a Rankine-Hugoniot relation with heat release gives the pressure in Region 5. The deflagration solution branch is chosen here. Though not important in determining the net force, it is also possible to choose an oblique shock location such that the flow in Region 6 is in the x direction only.

Thermal Explosion Theory

Thermal explosion theory provides an estimate for the flame sheet location. With the assumption of chemical reaction occuring in a fixed volume, well-stirred reactor with zero fluid velocity, Equations (1-6) are suitable to determine a time when the reaction rate becomes unbounded. This induction time is a function of the shocked fluid state and kinetic parameters. The shocked fluid velocity is used to associate an induction distance with the induction time. The flame sheet is fixed at this induction distance, which is measured along the wedge surface.

With the assumption of a static fluid $v_i = 0$, Eq. (1) holds that the fluid is incompressible and Eq. (2) holds that the P is only a function of time. Using the state relation (6), the system reduces to two equations in P and λ:

$$\frac{dP}{dt} = \frac{(\gamma-1)\rho\kappa q(1-\lambda)}{M_0^2}\exp\left(\frac{-\Theta}{M_0^2 T}\right) \tag{9}$$

$$\frac{d\lambda}{dt} = \kappa\,(1-\lambda)\exp\left(\frac{-\Theta}{M_0^2 T}\right), \tag{10}$$

$$P(0) = P_2, \qquad \lambda(0) = 0.$$

Here, the initial condition on pressure is given by the shock pressure, and the reaction progress is initially zero. These equations can be linearized by assuming $P = P_2 + P'$, $\lambda = \lambda'$, where the primed quantities are relatively small. The linearized equation can be solved exactly for the pressure perturbation P':

$$P' = -\frac{M_0^2 P_s}{\Theta \rho_s} \times \ln\left[1 - \frac{\Theta {\rho_s}^2}{M_0^4 {P_s}^2}\left(\gamma - 1\right) q\kappa \left(\exp \frac{-\Theta \rho_s}{M_0^2 P_s}\right) t\right]. \quad (11)$$

The pressure perturbation becomes unbounded as the argument of the logarithm approaches zero. This condition fixes a time, t_{ind}, when the reaction rate becomes $O(1)$ and gives rise to a thermal explosion. The induction time and distance, D_{ind}, are given by

$$t_{ind} = \frac{M_0^4 {P_2}^2}{\left(\gamma - 1\right) \Theta {\rho_2}^2 q\kappa} \exp\left[\frac{\Theta \rho_2}{M_0^2 P_2}\right] \quad (12)$$

$$D_{ind} = t_{ind}\sqrt{{u_2}^2 + {v_2}^2}, \quad (13)$$

where u_2 and v_2 are the x and y components of velocity behind the lead shock, respectively. The scaling chosen in this problem makes it difficult to see from Eq. (13) the behavior of the induction zone distance with increasing M_0, as M_0 is implicit in many variables. Calculations show that in general the induction zone length decreases with increasing M_0.

Results

We hold our upstream ambient fluid properties, except for incoming Mach number and heat release, constant. Our scaling, however, involves M_0; consequently, the dimensionless parameters vary from test to test. The ambient conditions which are held constant are $\tilde{P}_0 = 1.015 \times 10^5\ Pa$, $\tilde{\rho}_0 = 1.225\ kg/m^3$, $\gamma = 7/5$, $\tilde{k} = 1 \times 10^7\ s^{-1}$, $\tilde{E} = 1.019 \times 10^6\ J/kg$, $\theta = 5°$, $L = 0.1\ m$. Several values of heat release are chosen within the range $1.276 \times 10^6\ J/kg \leq \tilde{q} \leq 1.326 \times 10^6\ J/kg$. These values were chosen not so much to model a real system but so that the method could be successfully illustrated. Our values can be compared to typical values for $H_2 - O_2$ systems. The terms $\tilde{P}_0$, $\tilde{\rho}_0$, and γ are representative of a diatomic gas at atmospheric conditions. The heat release roughly corresponds to that of a lean $H_2 - O_2$ mixture with equivalance ratio ϕ in the range of $0.0477 \leq \phi \leq 0.0492$. This particularly narrow range of ϕ was chosen to illustrate an interesting bifurcation phenomena. It is possible to obtain less interesting solutions for broader ranges of equivalance ratios. The activation energy is of the same order of

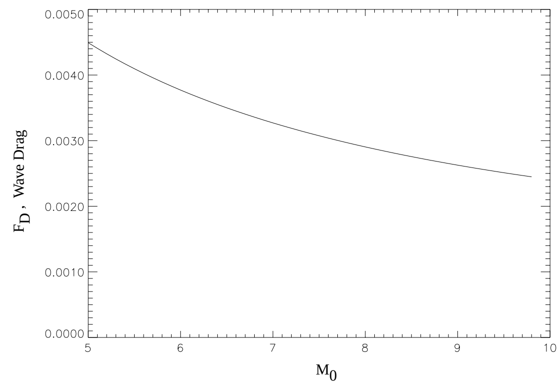

Figure 3: Wave drag force versus incoming Mach number

magnitude of those found in $H_2 - O_2$ reaction steps. The dimensionless activation energy Θ is 12.3, which suggests the high activation energy limit is appropriate. For our one-step first-order reaction kinetic model, it is difficult to compare our value of $\tilde{k}$ to typical models used in $H_2 - O_2$ combustion as different assumptions on the order of reaction are made for each step.

The projectile will have a steady velocity when the force due to pressure wave drag which tends to retard the motion is balanced by forces induced by combustion which tend to accelerate the projectile. The wave drag per unit depth F_D, defined to be positive in the positive x direction, was determined as a function of M_0 by applying the oblique shock and Prandtl-Meyer relations for zero heat release. The force was determined by integrating the pressure over the wedge area and is given by

$$F_D = P_4 \sin\theta \left(1/2\cos\theta\right) - P_2 \sin\theta \left(1/2\cos\theta\right). \quad (14)$$

The result is shown in Figure 3. The characteristic force per unit depth used for scaling is $P_0 M_0^2 L$. Because of this scaling, the scaled drag force drops with M_0, while the magnitude of the dimensional drag force increases with M_0.

When combustion is allowed, the net thrust force can be computed. The net thrust force per unit depth, F_{net}, defined to be positive if pointing in the negative x direction and is given by

$$F_{net} = P_4 \sin\theta \left(D_{ind} - 1/2\cos\theta\right) + P_5 \sin\theta \left(1/\cos\theta - D_{ind}\right) - P_2 \sin\theta \left(1/2\cos\theta\right). \quad (15)$$

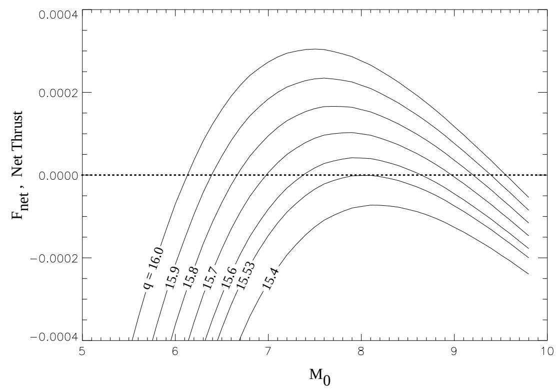

Figure 4: Net thrust force versus Mach number, varying heat release

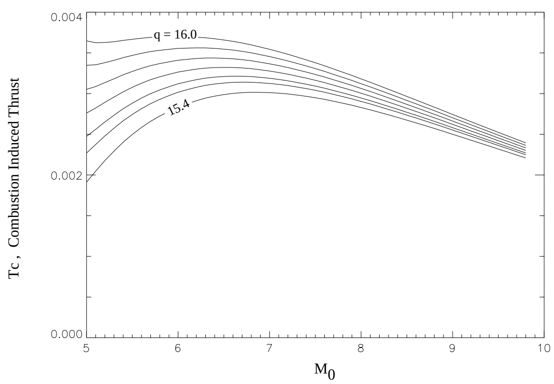

Figure 5: Combustion-induced thrust versus Mach number, varying heat release

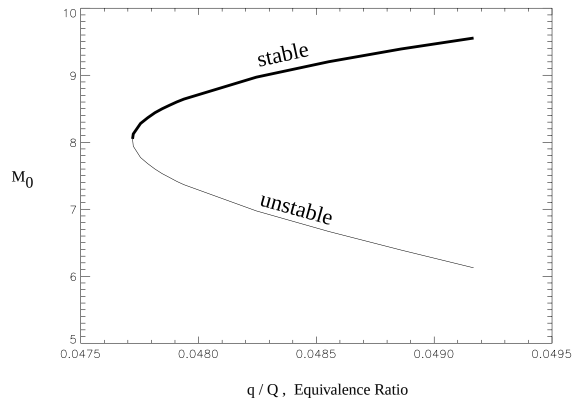

Figure 6: Bifurcation diagram for steady state speed versus equivalance ratio

Figure 4 shows F_{net} versus M_0 for several values of heat release. For low heat release the net thrust force is negative; the thrust force induced by combustion is not sufficient to overcome the wave drag. The combustion-induced thrust, F_c, is shown in Figure 5.

At a critical value of heat release, $q = 15.53$, there is a balance of combustion-induced thrust and drag such that the net thrust is zero. This occurs at a Mach number of 8.05. As heat release continues to increase, there are two distinct Mach numbers for which there is no net thrust. A perturbation in the Mach number for the steady solution at the lower Mach number results in a perturbed net force which tends to accelerate the projectile away from the equilibrium Mach number. Consequently, this is an unstable equilibrium. In the same manner, it is easily seen that the equilibrium solution at the higher Mach number is stable to such perturbations. As heat release is increased, the stable, high Mach number solution's Mach number increases and the flame sheet is located closer to the expansion fan, while the unstable, low Mach number solution's Mach number decreases and the flame sheet is located closer to the trailing edge.

The results are summarized on the bifurcation diagram in Figure 6. Here we plot the equilibrium Mach numbers versus equivalance ratio, q/Q. Here the value of Q is appropriate for stoichiometric $H_2 - O_2$ combustion and has a dimensionless value of 325.42. The lower branch is unstable while the upper branch is stable. The solutions shown correspond to stable flight speeds in the range of $2,700\ m/s - 3,200\ m/s$.

Conclusions and Recommendations

This study has shown the importance of the interaction of kinetic length scales with geometric length scales in determining steady propagation velocities for high Mach number propulsion devices. In particular, the Chapman-Jouguet velocity does not enter into this calculation.

This work can be used to guide full-scale numerical studies of similar problems for real materials. We recommend this in order to better understand the full capabilities of the ram accelerator or oblique detonation wave engine.

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