

ANALYSIS OF DETONATION STATES WHEN SHOCKING TWO-PHASE REACTIVE SOLIDS*

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ABSTRACT

Steady solutions to a set of two-phase reactive flow model equations are studied to test the hypothesis that observed deviations from Chapman-Jouguet detonation states in porous solid propellants are manifestations of the two-phase nature of the flow. Shock jump relations are presented. A simple expression for a minimum detonation wave speed analogous to a Chapman-Jouguet detonation for a single phase is given. The analogous Chapman-Jouguet point is a sonic point. In the appropriate limit the minimum detonation velocity varies linearly with initial bulk density, and the corresponding detonation pressure varies with the square of initial bulk density. Non-ideal gas effects play an important role in determining Chapman-Jouguet conditions.

I. INTRODUCTION

Detonation, a shock-induced chemical reaction, has been described by the theory of Chapman and Jouguet (CJ). The simple CJ theory analyzes a detonation as a jump discontinuity and solves algebraic equations that represent the conservation of mass, momentum, and energy for the detonation end states (pressure, density, etc.) as functions of the detonation velocity. In particular, the CJ theory identifies a unique wave speed and sonic end state for an unsupported detonation. In addition to end state analysis the structure of the detonation (i.e. the distribution of pressure, density, etc. in the reaction zone behind the wave head) must be considered when classifying possible steady solutions. In some cases certain end states can be ruled out because the end state cannot be attained from the initial state at the wave head. (The discussion of detonation structure is the topic of ZND theory.)

States and wave speeds other than those predicted by CJ theory may be realized. Under appropriate circumstances it can be shown for one-phase models that a so-called weak detonation (which travels supersonically relative to the detonation products) is possible. As discussed in ref. [1], weak detonations are observed in experiments. Also the numerical work of Butler, Lembeck, and Krier [2] suggests that both CJ and weak detonations may be admitted by equations of two-phase reactive flow used to model solid propellants and granulated explosives.

The evaluation of numerical solutions to the two-phase reactive flow equations (the work in ref. [2] is an example) requires a systematic study of the steady solutions. Knowledge of the one-dimensional steady solutions is necessary in order to develop the stability theory for two-phase detonation models. Steady solutions are also necessary to verify that numerical predictions of apparently steady solutions by unsteady model equations are not numerical artifices.

Our work analyzes a two-phase, unsteady detonation model similar to Butler and Krier's [3] and Baer, et. al. [4]. The first step in a complete structure analysis is the identification of detonation shock states and detonation end states. We first discuss shock jumps admitted by the equations. We then consider all possible end states admitted by the governing equations and, in particular, identify the minimum detonation wave speed admitted by our two-phase model. In this process we identify a CJ wave speed analogous to the CJ wave speed of the simple one-phase theory. These steps require the analysis of algebraic conditions. The additional constraints that a physically admissible detonation structure place on this problem and the identification of other admissible detonation states will be published in the future [5].

II. THE UNSTEADY TWO-PHASE MODEL

Here a two-phase model is proposed. The model is similar to models used by Butler and Krier [3] and Baer et. al. [4]. The proposed model utilizes simpler drag and heat transfer relations and a simpler solid equation of state in order to facilitate the analysis. Details about the rationale for the model in the context of continuum mixture theory can be found in refs. [6], [7], and [8]. For the proposed model it is assumed that each phase is a continuum; consequently, partial differential equations resembling one-phase equations are written to describe the evolution of mass, momentum, and energy in each constituent. In addition, each phase is described by a thermal state relation and a corresponding caloric state relation. One phase is assumed to be a solid, the other, gas. In order to close the system, a compaction equation similar to that of ref. [4] is adopted. As explained in ref. [4], the compaction law is suggested by an assumed form of the second law of thermodynamics. Choosing such a compaction equation also insures that the characteristics are real; thus, the initial value problem is well-posed. We emphasize that the choices we have adopted for the closure problem place a premium on simplicity so that explicit analytic calculations can be made whenever possible. At the same time the model adopted here is thought to be representative of a wider class of two-phase detonation models currently in use.

In order to describe the transfer of mass, momentum, and energy from one phase to another, phase interaction terms must be specified. The solid phase is assumed to be composed of spherical particles so that certain empirical

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correlations can be invoked [3], [9], [10]. These correlations describe the burning of spherical particles, particle-gas drag, and particle-gas heat transfer. Furthermore, it is assumed that the particles do not agglomerate or disintegrate; their number remains constant.

The unsteady equations are

$$\frac{\partial}{\partial t} [\rho_1 \phi_1] + \frac{\partial}{\partial x} [\rho_1 \phi_1 u_1] = \left(\frac{3}{r}\right) \rho_2 \phi_2 a P_1^m \quad (2.1)$$

$$\frac{\partial}{\partial t} [\rho_2 \phi_2] + \frac{\partial}{\partial x} [\rho_2 \phi_2 u_2] = -\left(\frac{3}{r}\right) \rho_2 \phi_2 a P_1^m \quad (2.2)$$

$$\frac{\partial}{\partial t} [\rho_1 \phi_1 u_1] + \frac{\partial}{\partial x} [P_1 \phi_1 + \rho_1 \phi_1 u_1^2] = u_2 \left(\frac{3}{r}\right) \rho_2 \phi_2 a P_1^m - \beta \phi_2 \phi_1 (u_1 - u_2) \quad (2.3)$$

$$\frac{\partial}{\partial t} [\rho_2 \phi_2 u_2] + \frac{\partial}{\partial x} [P_2 \phi_2 + \rho_2 \phi_2 u_2^2] = -u_2 \left(\frac{3}{r}\right) \rho_2 \phi_2 a P_1^m + \beta \phi_2 \phi_1 (u_1 - u_2) \quad (2.4)$$

$$\begin{aligned} & \frac{\partial}{\partial t} \left[\rho_1 \phi_1 \left(e_1 + \frac{u_1^2}{2} \right) \right] + \frac{\partial}{\partial x} \left[\rho_1 \phi_1 u_1 \left(e_1 + \frac{u_1^2}{2} + P_1 / \rho_1 \right) \right] = \\ & \left(e_2 + \frac{u_2^2}{2} \right) \left(\frac{3}{r} \right) \rho_2 \phi_2 a P_1^m - \beta \phi_1 \phi_2 u_2 (u_1 - u_2) - h \phi_1 \phi_2 (T_1 - T_2) \end{aligned} \quad (2.5)$$

$$\begin{aligned} & \frac{\partial}{\partial t} \left[\rho_2 \phi_2 \left(e_2 + \frac{u_2^2}{2} \right) \right] + \frac{\partial}{\partial x} \left[\rho_2 \phi_2 u_2 \left(e_2 + \frac{u_2^2}{2} + P_2 / \rho_2 \right) \right] = \\ & - \left(e_2 + \frac{u_2^2}{2} \right) \left(\frac{3}{r} \right) \rho_2 \phi_2 a P_1^m + \beta \phi_1 \phi_2 u_2 (u_1 - u_2) + h \phi_1 \phi_2 (T_1 - T_2) \end{aligned} \quad (2.6)$$

$$\frac{\partial}{\partial t} \left[\phi_2 / r^3 \right] + \frac{\partial}{\partial x} \left[u_2 \phi_2 / r^3 \right] = 0 \quad (2.7)$$

$$\frac{\partial \phi_2}{\partial t} + u_2 \frac{\partial \phi_2}{\partial x} = \frac{\phi_1 \phi_2}{\mu_c} [P_2 - P_1 - G \phi_2] - \alpha \left(\frac{3}{r} \right) \phi_2 a P_1^m \quad (2.8)$$

$$P_1 = \rho_1 R T_1 (1 + b \rho_1) \quad (2.9)$$

$$e_1 = c_{v1} T_1 \quad (2.10)$$

$$P_2 = (\gamma_2 - 1) c_{v2} \rho_2 T_2 - \rho_{20} c_{20}^2 \quad (2.11)$$

$$e_2 = c_{v2} T_2 + q \quad (2.12)$$

$$\phi_1 + \phi_2 = 1 \quad (2.13)$$

Here the subscript "o" denotes the undisturbed condition, 1 denotes the gas phase; 2, solid phase; ρ , density; ϕ , volume fraction; u , velocity; r , solid particle radius; P , pressure; m , burn index; a , burn constant; β , drag parameter; e , internal energy; h , heat transfer coefficient; R , gas constant; b , covolume correction; c_v , constant volume specific heat; c_{20} , ambient solid sound speed; μ_c , compaction viscosity; γ_2 , solid equation of state parameter; α , a compaction equation constant; G , pore collapse parameter; and q , heat of reaction.

Undisturbed conditions are specified as

$$\rho_1 = \rho_{10} \quad , \quad \rho_2 = \rho_{20} \quad , \quad \phi_1 = \phi_{10} \quad , \quad u_1 = 0,$$

$$u_2 = 0 \quad , \quad r = r_0 \quad , \quad T_1 = T_0 \quad , \quad T_2 = T_0$$

Equations (2.1,2) describe the evolution of each phase's mass; equations (2.3,4), momentum evolution; and equations (2.5,6), energy evolution. Here, a principle of two-phase modeling has been employed; the motion of an isolated constituent is determined by the action of other constituents on it. Homogeneous mixture equations are formed by adding equations (2.1) and (2.2), (2.3) and (2.4), and (2.5) and (2.6). Thus for the mixture, conservation of mass, momentum, and energy is maintained.

Inhomogeneities in equations (2.1-6) model interphase momentum, energy, and mass transfer. The model incorporates momentum transfer in the form of Stokes drag and energy transfer as constant-coefficient convective heat transfer. For mass transfer we attempt to utilize an empirical relation for the regression of particle radius. It is observed that the rate of change of particle radius is proportional to the surrounding pressure raised to some power. As we consider a constant number of compressible particles, our model must also allow the particle radius to change in response to a change in density of the particle phase. By combining the particle mass evolution equation (2.2) with the equation for number conservation (2.7), it is seen that the proposed model has the desired features:

$$\frac{\partial r}{\partial t} + u_2 \frac{\partial r}{\partial x} = -aP_1^m - \frac{r}{3\rho_2} \left(\frac{\partial \rho_2}{\partial t} + u_2 \frac{\partial \rho_2}{\partial x} \right) \quad (2.14)$$

that is, as the motion of a particle is followed, its radius may change in response to both burning and compression.

The compaction law is expressed in equation (2.8). Constituent 1 is a gas described by a virial equation of state (2.9). Likewise, constituent 2 is a solid described by a Tait equation of state (2.11). Assumption of a constant specific heat at constant volume for each phase allows caloric equations (2.10,12) consistent with the assumptions of classical thermodynamics to be written for each phase. The variable ϕ is defined as a volume fraction, $\phi \equiv$ constituent volume/total volume. All volume is occupied by constituent 1 or 2; no voids are permitted. This is enforced by equation (2.13). By using standard techniques [11], it can be shown that the characteristic wave speeds are $u_1, u_2, u_1 \pm c_1$, and $u_2 \pm c_2$, where c_i represents the sound speed of each phase. The uncoupling of the characteristic wave speeds is a consequence of the assumed form of the compaction law. Other closure relations will, in general, couple the characteristic wave speeds.

III. STEADY STATE ANALYSIS

Steady state analysis of two-phase equations has been performed by Kuo and Summerfield [12], [13], Krier and Mozaffarian [14], Taylor [15], and Drew [16]. The studies of both Kuo and Summerfield, and Drew only considered the deflagration of incompressible particles. Krier and Mozaffarian numerically studied two-phase detonation reaction zones with the assumption of incompressible particles. As discussed in ref. [17], for typical condensed phase detonation pressures, this assumption is unrealistic. Others such as Taylor [15] studied shock jumps in two-phase reactive flow equations. Here, we consider a steady two-phase detonation model with compressible particles. The results we present of two-phase detonation end states are new to the best of our knowledge.

Equations (2.1-13) can be re-cast in a more tractable form. First we write the equations in dimensionless form. For a right-running steady wave we make the Galilean transformation $\xi = x - Dt$, $v = u - D$ under which equations (2.1-8) collapse to eight ordinary differential equations. Next, equations (2.1), (2.3), and (2.5) may be eliminated in favor of homogeneous mixture equations formed by the addition of the steady form of equations (2.1) and (2.2), (2.3) and (2.4), and (2.5) and (2.6). The resultant mixture equations and the steady form of equation (2.7) may be integrated to form algebraic equations. Thus the steady two-phase model is described by four ordinary differential equations and nine algebraic equations.

DIMENSIONLESS MODEL EQUATIONS

Define dimensionless variables where "*" indicate a dimensionless quantity:

$$\xi_* = \xi / L \quad v_{*i} = v_i / D \quad P_{*i} = P_i / (\rho_{i0} D^2)$$

$$\rho_{*i} = \rho_i / \rho_{i0} \quad e_{*i} = e_i / D^2 \quad T_{*i} = c_{vi} T_i / D^2 \quad r_* = r / L$$

$$i = 1, 2$$

Here D is the wave speed; L , an undefined length scale to be associated with the reaction zone length; ρ_{i0} , the initial density of phase i ; c_{vi} , the specific heat at constant volume of phase i . Define the following independent dimensionless parameters as

$$\begin{aligned}\pi_1 &= 3a\rho_{10}^m D^{2m-1} & \pi_2 &= \beta L / (\rho_{20} D) & \pi_3 &= hL / (\rho_{20} c_{v1} D) \\ \pi_4 &= m & \pi_5 &= \rho_{10} / \rho_{20} & \pi_6 &= c_{v1} / c_{v2} & \pi_7 &= R / c_{v1} + 1 \\ \pi_8 &= c_{20}^2 / D^2 & \pi_9 &= \rho_{20} DL / \mu_c & \pi_{10} &= q / D^2 & \pi_{11} &= \phi_{10} \\ \pi_{12} &= r_0 / L & \pi_{13} &= b\rho_{10} & \pi_{14} &= c_{v2} T_0 / D^2 & \pi_{15} &= G / (\rho_{20} D^2) \\ \pi_{16} &= \alpha & \pi_{17} &= \gamma_2\end{aligned}$$

and the following dependent dimensionless parameters as

$$\begin{aligned}\pi_{18} &= \pi_{11} + \frac{1 - \pi_{11}}{\pi_5} \\ \pi_{19} &= \left[\pi_7 - 1 \right] \pi_6 \pi_{14} \left[1 + \pi_{13} \right] \\ \pi_{20} &= 1 - \pi_{11} \\ \pi_{21} &= \left[\pi_{17} - 1 \right] \pi_{14} - \pi_8 \\ \pi_{22} &= \pi_{11} \left[\pi_6 \pi_{14} + \frac{1}{2} + \pi_{19} \right] + \frac{1 - \pi_{11}}{\pi_5} \left[\pi_{14} + \pi_{10} + \frac{1}{2} + \pi_{21} \right] \\ \pi_{23} &= \pi_{11} \pi_{19} + \frac{1 - \pi_{11}}{\pi_5} \pi_{21}\end{aligned}$$

then the dimensionless model equations (for compact notation the stars are dropped) can be written economically as

$$\frac{d}{d\xi} \left[\rho_2 \phi_2 v_2 \right] = -\pi_1 \frac{\rho_2 \phi_2 P_1^{\pi_4}}{r} \quad (3.1)$$

$$\rho_2 \phi_2 v_2 \frac{dv_2}{d\xi} + \frac{d}{d\xi} \left[P_2 \phi_2 \right] = -\pi_2 \left[v_2 - v_1 \right] \phi_1 \phi_2 \quad (3.2)$$

$$\rho_2 v_2 \frac{de_2}{d\xi} + P_2 \frac{dv_2}{d\xi} = -\pi_3 \left[\pi_6 T_2 - T_1 \right] \phi_1 \quad (3.3)$$

$$v_2 \frac{d\phi_2}{d\xi} = \pi_9 \phi_1 \phi_2 \left[P_2 - \pi_5 P_1 - \pi_{15} \phi_2 \right] - \pi_1 \pi_{16} \frac{\phi_2 P_1^{\pi_4}}{r} \quad (3.4)$$

$$\rho_1 \phi_1 v_1 + \frac{1}{\pi_5} \rho_2 \phi_2 v_2 = -\pi_{18} \quad (3.5)$$

$$\rho_1 \phi_1 v_1^2 + P_1 \phi_1 + \frac{1}{\pi_5} \left[\rho_2 \phi_2 v_2^2 + P_2 \phi_2 \right] = \pi_{18} + \pi_{23} \quad (3.6)$$

$$\rho_1 \phi_1 v_1 \left[e_1 + \frac{v_1^2}{2} + \frac{P_1}{\rho_1} \right] + \frac{1}{\pi_5} \rho_2 \phi_2 v_2 \left[e_2 + \frac{v_2^2}{2} + \frac{P_2}{\rho_2} \right] = -\pi_{22} \quad (3.7)$$

$$r = \pi_{12} \sqrt[3]{\frac{-v_2 \phi_2}{1 - \pi_{11}}} \quad (3.8)$$

$$P_1 = [\pi_7 - 1] \rho_1 T_1 [1 + \pi_{13} \rho_1] \quad (3.9)$$

$$e_1 = T_1 \quad (3.10)$$

$$P_2 = [\pi_{17} - 1] \rho_2 T_2 - \pi_8 \quad (3.11)$$

$$e_2 = T_2 + \pi_{10} \quad (3.12)$$

$$\phi_1 + \phi_2 = 1 \quad (3.13)$$

Undisturbed conditions are

$$\rho_2 = 1 \quad v_2 = -1 \quad \phi_2 = \pi_{20} \quad T_2 = \pi_{14}$$

INERT SHOCK JUMP CONDITIONS

Equations (3.1-13) are valid throughout the steady flow field. If a detonation solution with a leading shock is assumed, it is necessary to know shock jump conditions. In order to form two-phase shock jump conditions, it is assumed that a thin shock wave exists in which no appreciable reaction, drag, heat transfer, or compaction occurs. Thus equations (3.1-4) may be replaced by the shock jump conditions:

$$\left[\rho_2 \phi_2 v_2 \right]_0^s = 0 \quad (3.14)$$

$$\left[P_2 \phi_2 + \rho_2 \phi_2 v_2^2 \right]_0^s = 0 \quad (3.15)$$

$$\left[\rho_2 \phi_2 v_2 \left(e_2 + v_2^2/2 + P_2/\rho_2 \right) \right]_0^s = 0 \quad (3.16)$$

$$\left[\phi_2 \right]_0^s = 0 \quad (3.17)$$

Here "s" denotes the shocked state and "0" the undisturbed state. Equations (3.14-17) and state relations (3.11,12) are sufficient to calculate the shock jumps for phase 2. The shock jumps for phase 1 are implied by the mixture equations (3.5-7) and state relations (3.9-10). Since porosity does not change through the shock, the shock jumps for the two phases are calculated independently.

Three solutions to the gas shock jump relations exist: the inert solution, an aphysical solution, and a shock solution. In the limit as π_{13} (or b) approaches zero, the aphysical solution predicts a gas density of $-1/\pi_{13}$ and is therefore rejected. In the same limit the shock solution can be thought of as a perturbation to the ideal gas shock solution. The asymptotic gas shock solution is

$$P_{1s} \equiv \left[\frac{2 - \pi_{14} \pi_6 (\pi_7 - 1)^2}{\pi_7 + 1} \right] - \left[\frac{2 + \pi_{14} \pi_6 (\pi_7 - 1)^2}{\pi_7 + 1} \right] \pi_{13} \quad (3.18)$$

For $\pi_{13} > 0$, the non ideal shock relations predict lower shock pressures than the ideal relation. Figure 1 shows a plot of the dimensional gas phase shock pressure versus the shock wavespeed. Unless otherwise noted, dimensional input parameters for this and all other cases are listed in Table I. It is thought that these input parameters are representative of a granular solid propellant. Nearly all parameters are identical to those listed in ref. [3].

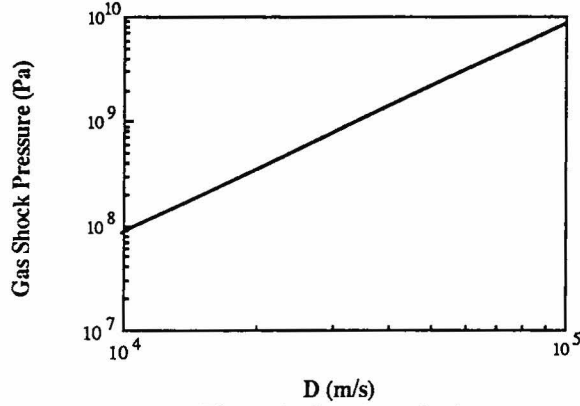


Figure 1. Gas phase shock pressure vs. shock wave speed

TABLE I

Dimensional Input Parameters

a	[m / (s*Pa)]	2.90×10^{-9}
ρ_{10}	[kg / m**3]	1.00×10^0
m		1.00×10^0
β	[kg / (s*m**3)]	1.00×10^4
ρ_{20}	[kg / m**3]	1.90×10^3
h	[J / (s*K*m**3)]	1.00×10^4
c_{v1}	[J / (kg*K)]	1.50×10^3
c_{v2}	[J / (kg*K)]	1.77×10^3
R	[J / (kg*K)]	3.75×10^2
c_{20}^2	[(m / s)**2]	5.00×10^5
q	[J / kg]	5.84×10^6
ϕ_{10}		3.00×10^{-1}
r_0	[m]	1.00×10^{-4}
b	[m**3 / kg]	4.00×10^{-3}
γ_2		3.00×10^0
G	[Pa]	2.00×10^6
α		1.00×10^0
μ_c	[kg / (m*s)]	2.00×10^{-6}
T_0	[K]	3.00×10^2

For the solid phase two solutions are admitted: the inert solution and the shock solution. The exact expression for solid phase shock pressure is

$$P_{2s} = \frac{2 - \pi_{14} (\pi_{17} - 1)^2}{\pi_{17} + 1} + \frac{\pi_{17} - 3}{\pi_{17} + 1} \pi_{18} \quad (3.19)$$

In figure 2, the dimensional solid phase shock pressure is plotted versus the shock wave speed.

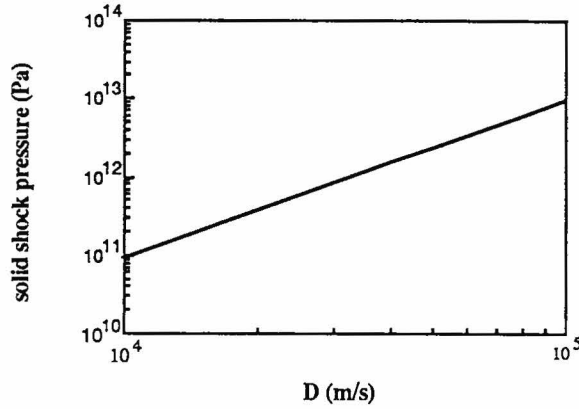


Figure 2. Solid phase shock pressure vs. shock wave speed

Similar relations are available for shock density, temperature, and energy. For the case of two ideal gases ($\pi_{13} = \pi_8 \equiv 0$) these equations reduce to familiar shock relations in terms of the shock Mach number. For the ideal gases shock Mach numbers may be expressed as

$$M_{1s}^2 = \frac{1}{\pi_7(\pi_7 - 1)\pi_{14}\pi_6}, \quad M_{2s}^2 = \frac{1}{\pi_{17}(\pi_{17} - 1)\pi_{14}} \quad (3.20-21)$$

END STATE ANALYSIS

An end state has been reached when at a given location ξ , the inhomogeneities of equations (3.1-4) are identically zero. The only physically viable end state admitted by (3.1-4) corresponds to a complete reaction ($\phi_2 = 0$) and temperature equilibrium ($T_1 = \pi_6 T_2$). In this state the mixture equations (3.5-7) allow for the end state to be determined. For $\phi_2 = 0$ ($\phi_1 = 1$), equations (3.5-7) can be combined to form an equivalent two-phase Rayleigh line (3.22) and two-phase Hugoniot (3.23)

$$P_1 - \pi_{23} = \pi_{18}^2 \left(1/\pi_{18} - 1/\rho_1 \right) \quad (3.22)$$

$$\left[e_1 - \frac{\pi_{11}\pi_6\pi_{14}}{\pi_{18}} \right] + \frac{1}{2} [P_1 + \pi_{23}] \left[1/\rho_1 - 1/\pi_{18} \right] = \frac{(1 - \pi_{11})(\pi_{14} + \pi_{10})}{\pi_5\pi_{18}} \quad (3.23)$$

From the state relations (3.9,10) we can write $e_1 = e_1(P_1, \rho_1)$ which is substituted into the Hugoniot equation (3.23). The Rayleigh line equation (3.22) allows ρ_1 to be eliminated in favor of P_1 . Substituting this in the Hugoniot equation (3.23) results in a cubic equation for P_1 . Three cases are possible: three distinct real solutions, two equal real solutions and a third real solution, and a real solution and a complex conjugate pair of solutions. When three distinct real solutions exist, two are analogous to the weak and strong solutions predicted by the simple one-phase theory. The third solution has no such counterpart and often is an aphysical solution with $\rho_1 < 0$. The third solution has not been thoroughly studied.

By imposing the condition that two real roots are degenerate (which forces the Rayleigh line and Hugoniot to be tangent) a minimum detonation velocity can be found. We will call this condition the CJ condition. Because the detonation velocity D is embedded in the dimensionless parameters, it is convenient to return to dimensional variables to express CJ conditions. Define the bulk density, bulk pressure, and bulk internal energy:

$$\rho_a = \rho_{10}\phi_{10} + \rho_{20}\phi_{20}, \quad P_a = P_{10}\phi_{10} + P_{20}\phi_{20}$$

$$e_a = \frac{\rho_{10}\phi_{10}e_{10} + \rho_{20}\phi_{20}e_{20}}{\rho_{10}\phi_{10} + \rho_{20}\phi_{20}}$$

By applying the tangency condition, it can be shown that in the limit as $b\rho_a$ and $P_a / (\rho_a e_a)$ approach zero that

$$D_{CJ} \equiv \frac{\sqrt{2e_a R (R + 2c_{v1})}}{c_{v1}} \left(1 + b\rho_a - \frac{c_{v1}^2}{2R (R + 2c_{v1})} \frac{P_a}{\rho_a e_a} \right) \quad (3.24)$$

$$P_{CJ} \equiv \frac{2e_a R}{c_{v1}} \rho_a \left(1 + b\rho_a - \frac{c_{v1}^2}{2R (R + 2c_{v1})} \frac{P_a}{\rho_a e_a} \right) \quad (3.25)$$

Similar expressions can be obtained for CJ density, temperature, and energy.

For $P_a = b = 0$ our formulae show that it is appropriate to treat the two-phase CJ condition as a one-phase CJ condition using ρ_a and e_a as effective one phase properties. Fickett and Davis [1] give equations for one-phase CJ properties. In these equations, one can simply substitute the bulk density for the initial density and the bulk internal energy for the chemical energy to obtain the two-phase CJ equations.

From equations (3.24,25) the CJ velocity depends linearly on ρ_a , the bulk density, and the CJ pressure varies with the square of bulk density in agreement with the trends discussed by Johansson and Persson [18]. This dependence is demonstrated in figures 3 and 4. These plots were obtained by solving the full set of non-linear equations to determine the CJ state. The bulk density was varied by varying the initial porosity.

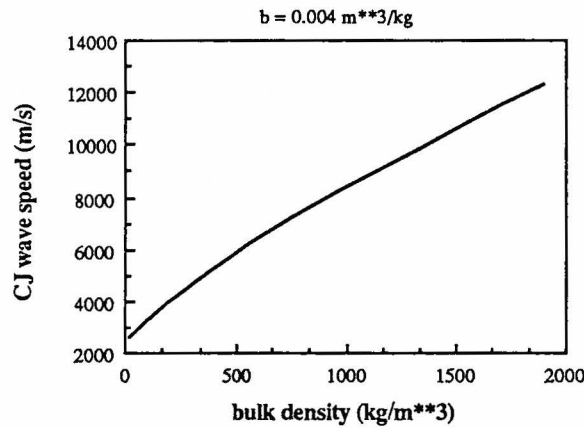


Figure 3. Minimum detonation velocity vs. bulk density for a non-ideal gas

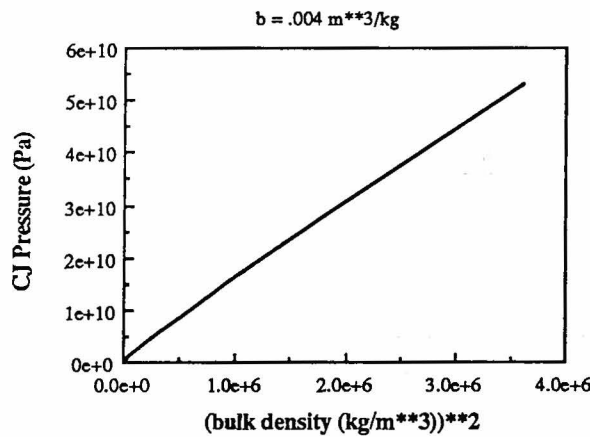


Figure 4. CJ pressure vs. square of bulk density for a non-ideal gas

Equations (3.24,25) indicate that the CJ state is quite sensitive to the non-ideal parameter b . In particular, it suggests that when the dimensionless group bp_a is of order 1, that non-ideal effects become quite important. This is demonstrated in figures 5 and 6 which for constant bulk density plot CJ wave speed and pressure versus the non-ideal parameter b . These figures were obtained by solving the full non-linear equations. In other works [3], the parameter b has been treated as adjustable in order to match CJ data.

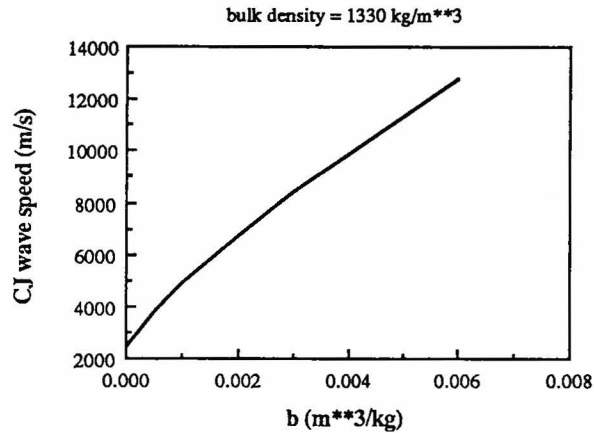


Figure 5. Minimum detonation wavespeed vs. non-ideal gas parameter b with constant bulk density

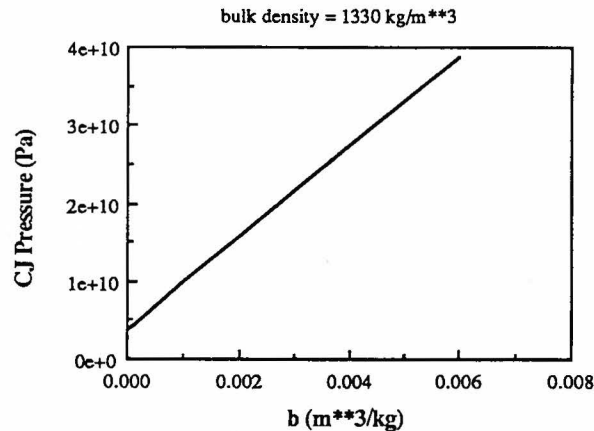


Figure 6. CJ pressure vs. non-ideal gas parameter b with constant bulk density

By numerically studying exact two-phase CJ conditions, it can be inferred that the CJ point is a sonic point, that is the gas velocity is equal to the local sonic velocity. In addition, numerical studies indicate that for $D > D_{CJ}$ the non-ideal strong point is subsonic while the non-ideal weak point is supersonic. This corresponds to the results of the simple one-phase theory.

It is important to note that the two-phase CJ solution is not necessarily admitted by our model equations when detonation structure is considered. A global phase space analysis of the solutions to the full set of differential-algebraic equations will determine which wave speeds correspond to physically admissible solutions.

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