OBLIQUE DETONATIONS: THEORY AND PROPULSION APPLICATIONS

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ABSTRACT

The oblique detonation, a combustion process initiated by an oblique shock, arises in most supersonic combustion applications including, most notably, the ram accelerator and the oblique detonation wave engine. Additionally, it is the generic two-dimensional compressible shocked reacting flow; consequently, its basic research value is inherent. The outstanding theoretical questions are also the fundamental practical questions: *e.g.* what conditions are necessary for steady solutions, what is the dependency of the steady propagation speed on the ambient condition, what is the susceptibility of the system to instability, and what is the behavior of the system in unsteady operation. A related topic which transcends all questions is the ability to describe these phenomena computationally. At this early stage, these issues are most clearly addressed with simple models. This paper will review the application of such models to oblique detonations and discuss their future relevance.

1. Introduction

This paper will attempt to demonstrate the utility of modeling simple systems in order to gain understanding of both oblique detonations and their application to propulsion devices. The plan of this paper is as follows. After a short review, some basic research issues are outlined followed by a summary of possible modeling approaches. Next a simple model is proposed which is both amenable to analysis and in a certain sense representative of real propulsion

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adopted from J. Buckmaster, T. L. Jackson, and A. Kumar, 1994, *Combustion in High Speed Flows*, Kluwer: Dordrecht, pp. 345-371.

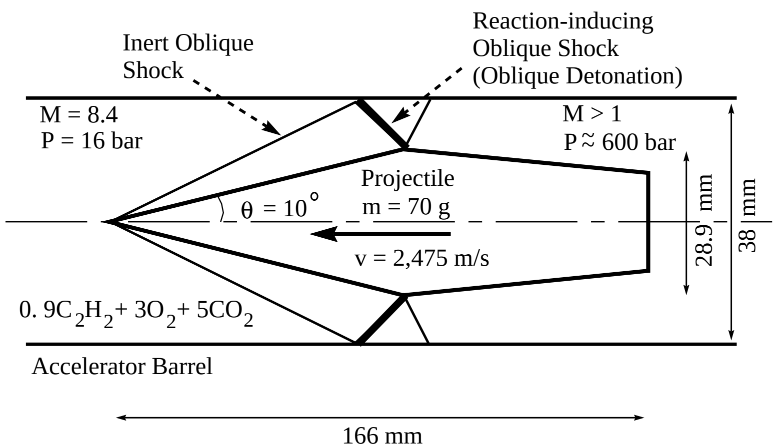

Figure 1: Schematic of ram accelerator, adopted from Hertzberg, *et al.*, 1991.

systems. A summary of the author's previous results in modeling aspects of this system is then presented. The paper concludes with recommendations for future studies.

1.1. Review

The oblique detonation has received attention of late because of its role as the primary combustion mechanism in certain high-speed propulsion applications. One such device is the ram accelerator, (see Figure 1) first tested in recent years (Hertzberg, *et al.*, 1988, 1991). In this application, a high speed projectile is fired at high velocity from a light gas gun into a tube filled with an unreacted mixture of combustible gases. Hertzberg, *et al.*, 1991, observed that upon entering a $16m$ length, $38mm$ bore tube filled in its first three stages with varying combinations of CH_4, O_2, N_2, and He at a pressure of $31bar$ and in its final stage with $0.9C_2H_4 + 3O_2 + 5CO_2$ at a pressure of $16bar$, that a shock-induced combustion process accelerates a $70g$ projectile from an initial velocity of near $1,200m/s$ to a velocity of $2,475m/s$ (corresponding to a Mach number, $M = 8.4$) at the end of the tube, at which location it was still accelerating. Downstream pressures in the neighborhood of $600bar$ are measured. The diameter of the main body of the projectile was $28.9mm$. Its length was $166mm$ and the leading edge conical half-angle $\theta = 10\,\text{deg}$. Four stabilizing fins (not shown) of diameter $38mm$ were a part of the aft-body. A portion of the oblique shock train is sketched in Figure 1; the various expansion fans and wave interactions are not included. Figure 1 depicts the first reflected shock triggering significant chemical reaction; the temperature-sensitive reaction would be associated with the lead shock for faster projectile speeds, and with a downstream shock for slower speeds. For even slower speeds, the reaction would be downstream of the projectile. It was suggested that such a device can be scaled for direct launch to orbit, for hypervelocity impact studies, and for a hypersonic test facility.

These experiments have directly motivated further, primarily numerical studies: (Brackett and Bogdanoff, 1989), (Bruckner, *et al.*, 1991), (Yungster, *et al.*, 1991), (Yungster and Bruckner, 1992), (Bogdanoff, 1992), Yungster (1992), and (Pepper and Brueckner, 1993). In particular, the numerical calculations of Yungster and Bruckner, 1992, predict that a ram accelerator can achieve a steady-state velocity such that a combustion-induced thrust force balances drag forces. This condition is achieved at a velocity near $9,600m/s$, corresponding to a Mach number near 12 for a mixture, initially at a pressure of $20.3bar$ and composed of $5H_2 + O_2 + 4He$, flowing over an axisymmetric projectile of half angle $\theta = 14\,\text{deg}$, diameter $29mm$, and overall length $190mm$ in a tube of diameter $38mm$. Direct comparisons cannot be made with the experiments of Hertzberg, *et al.*, as hydrogen rather than hydrocarbon combustion was modeled, because much higher flight velocities were modeled, and because the geometry, which contained an additional constant area mid-section, was slightly different.

Another relevant propulsion device is the proposed oblique detonation wave engine (ODWE). The idea of using an ODWE for supersonic combustion for a high-speed plane has existed for decades (*e.g.* Dunlap, *et al.*, 1958). The hypothesized operation is as follows (see Figure 2, adopted from Dunlap, *et al.*). Supersonic air enters the inlet. On-board fuel is injected downstream which mixes with the air without significant reaction. The mixture then encounters a downstream wedge. The oblique shock associated with the wedge compresses and ignites the mixture, generating a propulsive force. Relative to conventional air-breathing engines with subsonic combustion, Dunlap, *et al.* cite the ODWE's advantages as 1) simpler supersonic inlet diffuser design since the inherently supersonic oblique detonation does not require deceleration to a subsonic state, 2) re-

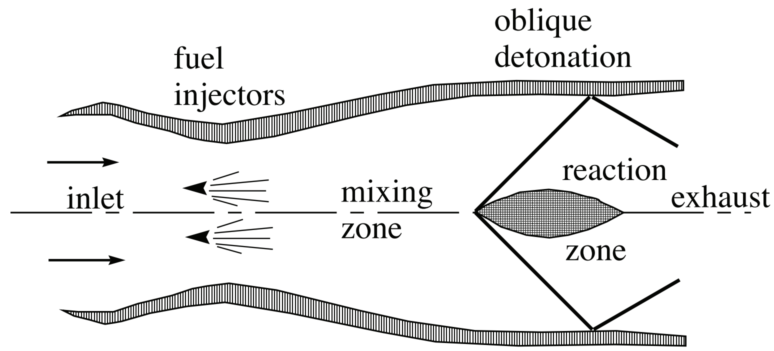

Figure 2: Envisioned oblique detonation wave engine, adopted from Dunlap, *et al.*, 1958.

duced total pressure losses, 3) shorter combustion chamber length, 4) no ignition device other than the wedge, and 5) faster flight velocities. Cited concerns are 1) the lack of static thrust, 2) uncertainty as to whether mixing lengths are practical, and 3) uncertainty with regards to the process's stability.

Renewed emphasis on high speed air-breathing propulsion alternatives led to modern studies of wedge-initiated ODWE's, (Cambier, *et al.*, 1989, 1990). Alternatively, laser-initiation has been studied (Carrier, *et al.*, 1992). Both wedge- and laser-initiated detonation engines contrast the more-studied Ferri engine in which two supersonic streams, one fuel and the other oxidizer, are brought together so that burning occurs in a convective-diffusive mixing layer.

Other more basic studies have relevance. Several give a Rankine-Hugoniot (RH) analysis of oblique detonations: *e.g.* (Siestrunck, *et al.*, 1953), (Larisch, 1959), (Gross, 1963), (Oppenheim, *et al.*, 1968), (Chernyi, 1969), (Buckmaster and Lee, 1990), and (Pratt, *et al.*, 1991). Other analyses are for either steady two-dimensional or unsteady one-dimensional flows with spatially resolved structure; many of these focus on the related topic of dissociation and vibrational relaxation: *e.g.* (Clarke, 1960), (Moore and Gibson, 1960), (Sedney, 1961), (Spence, 1961), (Vincenti, 1962), (Capiaux and Washington, 1963), (Lee, 1964), (Spurk, *et al.*, 1966), and (Fickett, 1984).

Recent analyses which this author and colleagues have performed have placed emphasis on oblique detonations with resolved reaction zone structure and the connections of these structures with the predictions of a RH analysis: (Powers and Stewart, 1992), (Powers and

Gonthier, 1992a), and (Grismer and Powers, 1992). In addition, there is a new large body of general unsteady analyses of one- and two-dimensional detonations which though germane, have largely not been applied in propulsion studies, *e.g.* (Bdzil and Stewart, 1986), (Stewart and Bdzil, 1988), (Buckmaster, 1990), (Clarke, *et al.*, 1990), (Lee and Stewart, 1990), (Jackson, *et al.*, 1990), (Lasseigne, *et al.*, 1991), (Bourlioux, *et al.*, 1991), (Bourlioux and Majda, 1992), and (Bdzil and Kapila, 1992). These studies build largely on the Zeldovich, von Neumann, Doering (ZND) theory which has undergone continuous refinement since being introduced in the 1940's; extensive reviews exist (Fickett and Davis, 1979).

Lastly, there exist fundamental experimental studies, *e.g.* (Gross and Chinitz, 1960), (Nicholls, 1963), (Rubins and Rhodes, 1963), (Behrens, *et al.*, 1965), (Strehlow, 1968), (Strehlow and Crooker, 1974), (Lehr, 1972), and (Liu, *et al.*, 1986). Of potential relevance, especially in light of Dunlap *et al.*'s concern, are dramatic observations of one- and three-dimensional detonation instabilities. One-dimensional instability can be observed when high speed projectiles are fired into reactive mixtures. An example is sketched in Figure 3, which is a representation of a photograph from Lehr. For this particular sketch, a $15mm$ diameter projectile travels into a mixture with composition $2H_2 + O_2 + 3.76N_2$ at a pressure of $0.427bar$ at an instantaneous velocity of $2,029m/s$ (corresponding to a Mach number of 5.04), slightly less than the Chapman-Jouguet (CJ) velocity of the mixture. The observed pulsations, which are at a frequency of $1.04MHz$, have been interpreted by Fickett and Davis as an essentially one-dimensional phenomena originating near the projectile tip that leaves traces which remain downstream. Evidence of three-dimensional detonation instability is given by Strehlow and Crooker. When the walls of a tube are coated with smoke, a detonation wave will sometimes leave a regular cell pattern on the walls. A sample pattern is sketched in Figure 4, which was traced from a photograph of Strehlow and Crooker. In this case the initial composition was $2H_2 + O_2 + 3Ar$ at a pressure of $0.077bar$. It is thought that the patterns are the result of a shock triple point leaving its trace on the coated wall. As with the unsteady analyses, the implications for high-speed propulsion of observed detonation instabilities have not been fully explored.

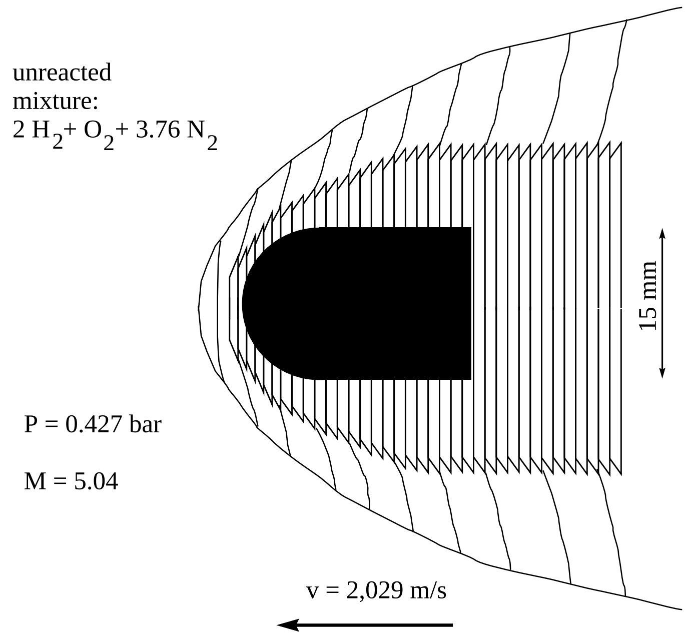

Figure 3: Sketch of observed combustion instability in projectile firing experiment, adopted from photographs of Lehr, 1972.

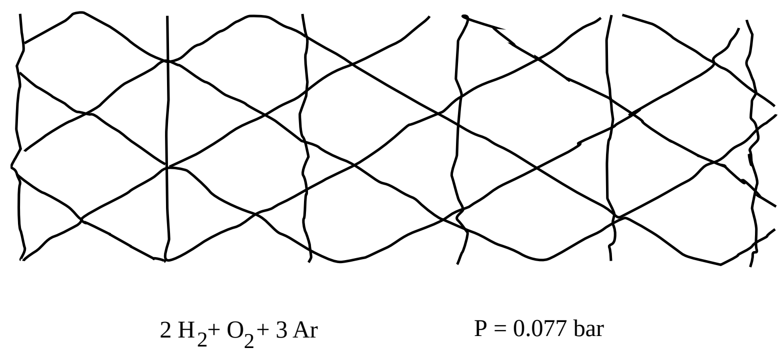

Figure 4: Sketch of observed patterns on smoke-coated foils after passage of an unstable detonation wave, adopted from photographs of Strehlow and Crooker, 1974.

1.2. Idealized Oblique Detonation Definition

Before discussing detailed results, it is useful to have a working definition of an oblique detonation. As reviewed by Pratt, *et al.*, this has been a controversial topic. Here a definition is proposed which has been suitable for our studies. We define an oblique detonation as a combustion process which is initiated by an oblique shock in a flow field in which the fluid properties vary within length scales dictated primarily by the rate of chemical reaction. In such a process the oblique shock raises the temperature appreciably but is sufficiently thin to prevent significant combustion within the shock. Past the shock, the higher fluid temperature allows for significant reaction to occur in a spatially resoved reaction layer. The definition has the advantage of being device-independent as it does not require geometric length scales.

Though other scenarios are possible, one typically considers the oblique shock to be generated by the supersonic flow over a geometrical obstacle. It is illustrative to consider the flow over a straight wedge of half angle θ and semi-infinite length to frame some important issues. For such a geometry, as depicted in Figure 5, the shock angle near the wedge tip before significant reaction has occurred is that of an inert oblique shock. Far from the wedge tip the shock angle relaxes to a constant value, greater than the inert value. Powers and Stewart's linear analysis for small heat release shows this change can be attributed to the net effect of downstream local heat release disturbances. Such disturbances are propagated along characteristics which, in a complex reflection process between the wedge and the shock, strengthen the lead shock causing its inclination angle to increase with increasing distance from the wedge tip. Consequently, a region of shock curvature exists near the wedge tip which induces vorticity which is convected along streamlines in a layer near the wedge surface. Far from the shock and wedge surface, the flow relaxes to an irrotational, equilibrium, uniform state.

Such a definition allows the oblique detonation to be thought of as the two-dimensional analog of the ZND model for one-dimensional detonations (Fickett and Davis). The ZND model describes a reaction zone structure which links a shocked state to one of the three states identified by a RH analysis: a subsonic state (strong), a sonic state (CJ), or a supersonic state (weak). Energy release in the subsonic region behind the lead shock serves to drive the wave forward.

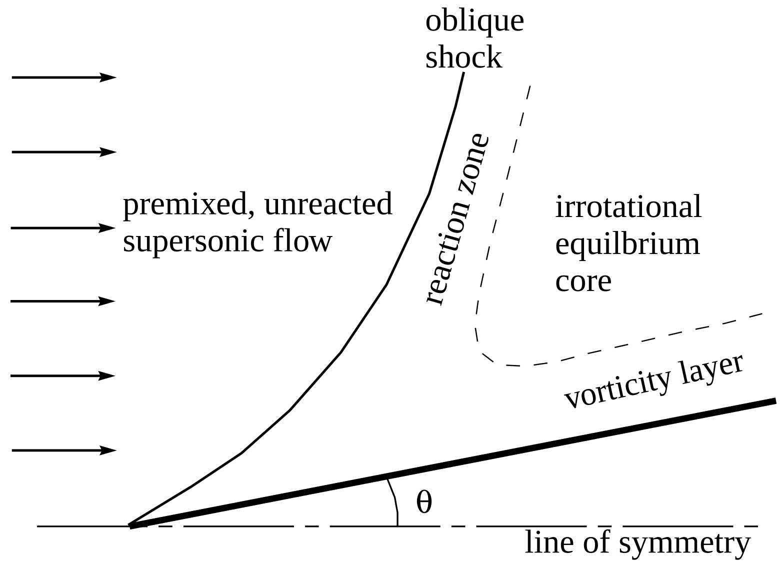

Figure 5: Straight wedge-curved shock oblique detonation configuration.

Oblique detonation analogs for each case exist, which are repeated later in this paper. Also, the one-dimensional steady ZND detonation is often considered in the context of a piston problem; for strong solutions a portion of the energy to drive the wave comes from the piston. For the oblique detonation of Figure 5, the wedge plays the role of the piston.

This definition is not universally accepted. In many oblique detonations the heat release only forms a small portion of the flow's total energy. In such case the wave is not primarily driven by the heat release; consequently, there is some reluctance to use the term "detonation." However, inasmuch as it is proper to describe a one-dimensional reactive wave driven by a supersonic piston as a strong (or overdriven) "detonation," it is proper to describe the corresponding two-dimensional waves as oblique "detonations." Also, as discussed by Pratt, *et al.*, distinctions are often drawn between an oblique detonation and "shock-induced combustion," in that the oblique detonation exists when the reaction occurs in a thin zone indistinguishable from an oblique shock, while the "shock-induced combustion" is characterized by an inert shock followed by a thick reaction zone. It is noted, however, that such a distinction requires the existence of an extraneous, independent, non-kinetic length scale, such as might be given by the diameter of a combustion chamber, in order to properly classify the phenomena. In addition, the distinction is inconsistent with the ZND characterization of a detonation as a shock followed by a resolved reaction zone.

1.3. Research Issues

The configuration sketched in Figure 5 has both basic and applied value. Most importantly, it captures the essence of two-dimensional shocked reactive flows. As such, it seems necessary that this flow should form the basis for comprehension of more complex matters. Many of the basic research issues which remain for this flow are also issues of practical concern. Outstanding questions include what conditions are necessary for a steady state solution, what is the susceptibility of steady solutions to instability, and what is the fully transient behavior. Such questions bear directly on the operating characteristics of any propulsion device. These issues have been addressed in the detonation literature primarily for one-dimensional flows; for two-dimensional flows, there are relatively fewer studies.

1.4. Modeling Approaches

A variety of modeling philosophies and techniques have been used to address these issues. One philosophy, the more Aristotelian, is to capture physical reality as much as possible. In ram accelerator or ODWE configurations, this typically involves modeling detailed geometries, detailed chemistry, diffusive transport, state dependent material properties, and turbulence. Such an approach, which necessitates a numerical solution, offers, significantly, the potential for predictions which closely mimic experiments to the extent that the computer becomes a substitute for the wind tunnel. As envisioned, all prototypes could be fully tested with the numerical model.

In the absence of verifying experiments or solutions from alternate techniques, caution must be used in this approach. First, many times the numerical results are as difficult to interpret as experiments because of the large number of simultaneously competing mechanisms. Moreover, it is often the case that the equations predict three-dimensional unsteady flows with phenomena occurring on widely varying scales. In combustion, scales are usually imposed by detailed kinetic models; time scales can range from $10^{-9}s$ to $10^{2}s$ and are typically far more severe than acoustic, diffusive, or turbulent scales (Maas and Pope, 1992). Capturing all of these scales can place severe demands on present computer resources. Additionally, the inherent non-linearity of the problem can give rise to a variety of coarse and fine scale structures. Striking evidence of these are given by the two-dimensional calculations of Bourlioux and Majda for a one-step reaction with Arrhenius kinetics. Furthermore, it can be shown (Yee, *et al.*, 1991, Lafon and Yee, 1992) that discretization can actually mask the true solution features, and thus, insofar as the model represents physical reality, mask the actual flow physics. These papers, both of which are specifically addressed to modeling issues in hypersonic propulsion, apply typical discretization techniques to equations with known analytical solutions. Many dangers are discussed including the possibility of prediction of instability for known stable solutions, prediction of stability for known unstable solutions, and convergence to incorrect equilibria.

An alternative philosophy, the more Platonic, is to seek a complete understanding of a few selected phenomena. Typically, details are sacrificed at the discretion of the modeler so as to get to the essence of the problem. For propulsion applications, this may mean modeling simple geometries, simple chemistry, inviscid fluids, constant properties, and no turbulence modeling. Solution techniques are more varied and involve such methods as nonlinear analysis of dynamic systems, asymptotic analysis, and the method of characteristics. Advantages of this approach are that causality is easier to establish, quick and useful estimates are often provided, and best- and worst-case scenarios can sometimes be formed. Significantly, exact and asymptotic solutions are sometimes available, rendering determination of parametric dependencies and optimization easier. Such solutions also provide valuable test cases for numerical methods designed to solve more complex problems. An obvious disadvantage is that predictions are often far from physical reality. The remainder of this paper gives pertinent examples of this approach.

2. Idealized propulsion configuration

The configuration of Figure 5 is well-suited to study oblique detonations. However, because combustion on only the front side of the wedge is modeled, this represents a case where the force generated by

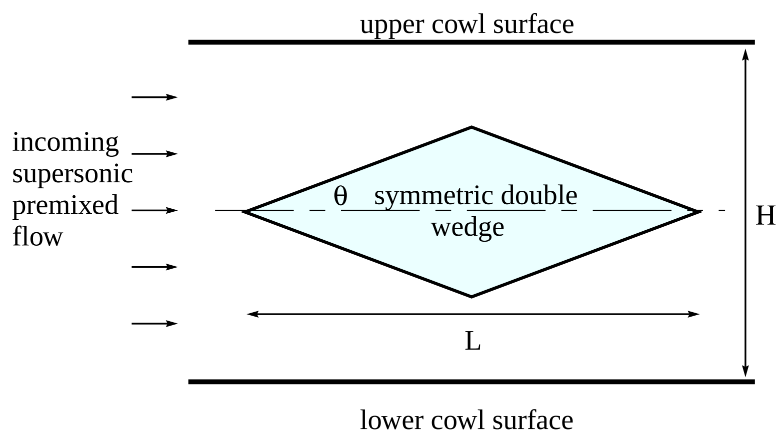

Figure 6: Schematic of idealized confined propulsion configuration, from Powers and Gonthier, 1992b.

combustion retards the body's motion. In order to achieve a propulsive force, one must consider the combustion over both sides of a finite projectile. It has been proposed (Powers and Gonthier, 1992b) to consider the geometry of Figure 6, similar to the geometry used by Yungster and Bruckner, but planar and with one fewer geometric length scale. Here a planar double wedge of half angle θ and length L is placed between confining walls separated by distance H. Such a geometry is representative of a ram accelerator if the confining walls are stationary and is representative of an ODWE if the confining walls move with the double wedge.

To further simplify, Powers and Gonthier, 1992b, only considered the limit $H \to \infty$, Figure 7. While this geometry retains at most a rudimentary resemblance to actual devices, it is both potentially propulsive and amenable to analysis. The incoming flow is considered to be supersonic, premixed, and unreacted. An oblique or bow shock will exist at or near the leading edge. The shock should be of sufficient strength to initiate the induction phase of chemical reaction but not so strong that the reaction occurs immediately. The flow will expand in a rarefaction fan at the projectile apex. It is important that there be sufficient heat release to prevent the reaction from being quenched by the rarefaction. On the lee side, significant combustion should occur so that a force to counterbalance wave drag is generated. Finally to turn the flow, a shock at the trailing edge is

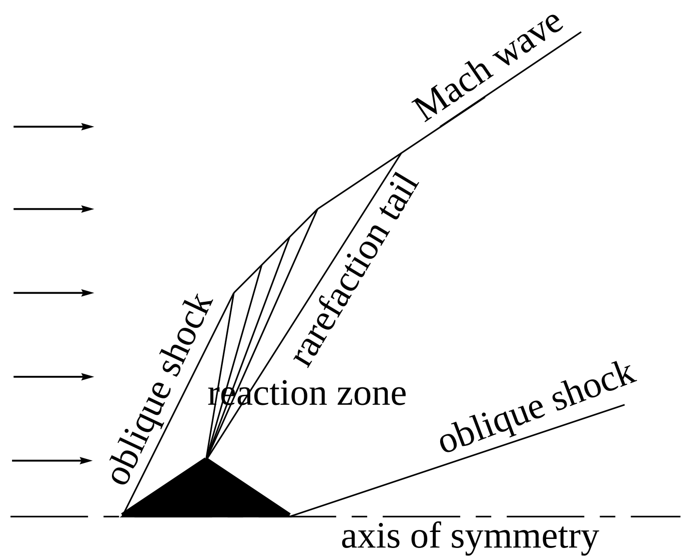

Figure 7: Schematic of idealized unconfined propulsion configuration, from Powers and Gonthier, 1992b.

required.

Of fundamental importance is the self-sustaining propagation velocity. Neglecting body forces, such a velocity is achieved when there is a balance of surface forces on the projectile, that is when the thrust force induced by combustion equals the wave and viscous drag forces. As a solution technique for any particular fluid and combustion model, one can select the steady wave speed by a trial and error process. One can then determine the parametric dependency of the steady wave speed on geometry and material properties and also figures of merit such as propulsive efficiency. The steady solution also serves as a base state for stability and unsteady analyses.

Many models can be used to address such questions. Next, a simple model used by the author will be presented and, as an example of such a model's utility, the author's oblique detonation predictions from this model will be summarized.

3. Idealized model

The model equations are taken to be the unsteady Euler equations and species evolution equation for a reactive calorically perfect ideal gas. These are expressed in dimensionless form as:

$$\frac{d\rho}{dt} + \rho\frac{\partial v_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{dv_i}{dt} + \frac{1}{\rho}\frac{\partial P}{\partial x_i} = 0, \tag{2}$$

$$\frac{de}{dt} + P\frac{\partial v_i}{\partial x_i} = 0, \tag{3}$$

$$\frac{d\lambda}{dt} = \kappa\,(1-\lambda)\exp\left(\frac{-\Theta}{M_0^2 T}\right), \tag{4}$$

$$e = \frac{1}{\gamma-1}\frac{P}{\rho} - \frac{\lambda q}{M_0^2}, \tag{5}$$

$$P = \rho T. \tag{6}$$

The variables contained in Equations (1–6) are the density ρ, the Cartesian velocity component v_i, the pressure P, the temperature T, the internal energy e, the reaction progress variable λ, and the Cartesian position coordinate x_i. Here the substantial derivative $\frac{d}{dt} = \frac{\partial}{\partial t} + v_i\frac{\partial}{\partial x_i}$ The freestream Mach number is M_0. Other dimensionless parameters include the ratio of specific heats γ, a kinetic parameter κ, the heat of reaction q, and the activation energy Θ. Equations (1–3) represent the conservation of mass, momenta, and energy, respectively. Equation (4) is a species evolution equation which incorporates an Arrhenius depletion model. Equations (5–6) are caloric and thermal equations of state. A single, first-order, irreversible, exothermic reaction is employed, $A \rightarrow B$. The reaction progress variable λ ranges from zero before reaction to unity at complete reaction. Species mass fractions, Y_i are related to the reaction progress variable by the formulae, $Y_A = 1 - \lambda$, $Y_B = \lambda$. Initial preshock conditions are specified as $\rho = 1$, $u = \sqrt{\gamma}$, $v = 0$, $P = 1/M_0^2$, and $\lambda = 0$.

Equations (1-6) have been scaled such that in the hypersonic limit ($M_0^2 \rightarrow \infty$) the pressure, density, and velocities are all $O(1)$ quantities behind the lead shock. The geometric length of the projectile (L) is chosen as the reference length scale. In terms of dimensional variables (indicated by the notation "˜") and dimensional pre-shock ambient conditions (indicated by the subscript "0"), the dimensionless variables are defined by

$$\rho = \frac{\tilde{\rho}}{\tilde{\rho}_0}, \qquad P = \frac{\tilde{P}}{M_0^2 \tilde{P}_0},$$

$$u = \frac{\tilde{u}}{M_0 \sqrt{\tilde{P}_0/\tilde{\rho}_0}}, \qquad v = \frac{\tilde{v}}{M_0 \sqrt{\tilde{P}_0/\tilde{\rho}_0}},$$

$$x = \frac{\tilde{x}}{L}, \qquad y = \frac{\tilde{y}}{L}. \tag{7}$$

Remaining dimensionless parameters are defined by the following relations:

$$q = \frac{\tilde{\rho}_0 \tilde{q}}{\tilde{P}_0}, \qquad \Theta = \frac{\tilde{\rho}_0 \tilde{E}}{\tilde{P}_0}, \qquad \kappa = \frac{\tilde{k}}{\frac{M_0}{L}\sqrt{\frac{P_0}{\rho_0}}}, \tag{8}$$

Here, $\tilde{E}$ is the dimensional activation energy, $\tilde{q}$ is the dimensional heat of reaction, and $\tilde{k}$ is the dimensional kinetic rate constant.

4. Summary of Results

To gain a basic understanding of two-dimensional reactive flows, and to gain insight into the possible propulsion situation in which the combustion is induced on the front side of the wedge, it is reasonable to study oblique detonations over semi-infinite wedges. In such a case, Equations (1-6) can be rescaled so that the kinetic rate defines the length scale. Figure 8 gives a diagram of a particularly simple type of such an oblique detonation. This was studied by Powers and Stewart for one-step kinetics [as in Equation (4)] and extended by Powers and Gonthier, 1992a, to two-step kinetics [not written explicitly in Equations (1-6)]. The flow considered is an incoming unreacted gaseous mixture at supersonic Mach number, $M_0 > 1$, which encounters a straight shock, inclined at angle β to the horizontal, which is attached to a curved wedge. The mixture reacts downstream of the shock in the reaction zone. The special case in which the flow has variation in the direction normal to the shock, taken to be the x direction, but no variation in the direction parallel to the shock, taken to be the y direction, was considered. The origin was taken to be the wedge tip. The streamlines were taken to form

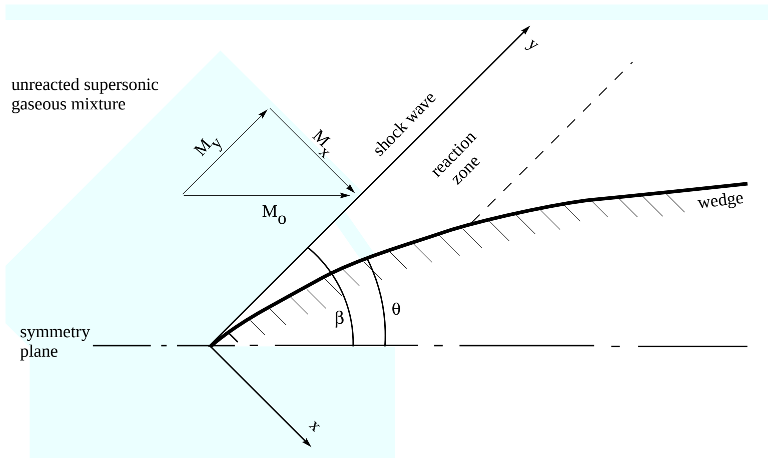

Figure 8: Curved wedge-straight shock oblique detonation configuration, from Powers and Gonthier, 1992a.

an angle θ with the horizontal. At complete reaction, θ relaxes to a constant value. The flow has symmetry about the horizontal plane.

A RH analysis has been commonly used to restrict the potential equilibrium states which may be obtained in an oblique detonation. The RH analysis allows determination of both β, θ shock and detonation polars. If the dimensionless instantaneous heat release for one-step kinetics, Q, is taken to be

$$Q = \lambda q, \tag{9}$$

then for $M_0 = 10$, $\gamma = 7/5$, and $q = 25$, Figure 9 shows such polars for an inert oblique shock, $\lambda = 0, Q = 0$, and a complete reaction oblique detonation, $\lambda = 1, Q = 25$.

Following Pratt, *et al.*, the final value of the Mach number normal to the shock, M_x, and analogies with inert oblique shock nomenclature were used to classify oblique detonations. For shock angles below a critical value $\beta < \beta_{CJ}$, there is no real solution to the RH equations. For $\beta = \beta_{CJ}$, there is one solution which corresponds to the CJ solution of one-dimensional theory. For $\beta = \beta_{CJ}$, at complete reaction the normal Mach number is sonic, $M_x = 1$. For $\beta > \beta_{CJ}$, two solutions are obtained. The solution corresponding to the smaller wedge angle has a supersonic normal Mach number, $M_x > 1$, at complete reaction and is known as a weak underdriven solution. Its counterpart with the higher wedge angle is known as

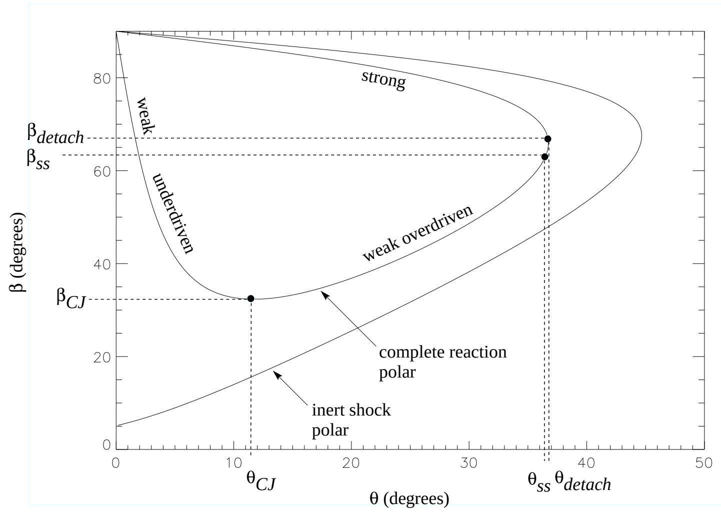

Figure 9: Inert ($Q = 0$ and complete reaction ($Q = 25$) shock polars, from Powers and Gonthier, 1992a.

a weak overdriven solution if $\beta < \beta_{detach}$ and a strong solution if $\beta \geq \beta_{detach}$. For both weak overdriven and strong solutions, the final normal Mach number is subsonic, $M_x < 1$. Here β_{detach} is the shock angle corresponding to the wedge angle θ_{detach} beyond which there is no attached shock solution. The nomenclature "weak" and "strong" is suggested by oblique shock theory and is not consistent with the nomenclature of one-dimensional detonation theory.

The two-dimensional steady flow can be further characterized by the hyperbolic or elliptic character of the governing partial differential equations. With the total Mach number M calculated from the velocity magnitude, the equations are elliptic if $M < 1$ and hyperbolic if $M > 1$. The subsonic to supersonic transition takes place at β_{SS} which is slightly less than β_{detach}. Strong solutions terminate at a subsonic point, $M < 1$. Weak overdriven solutions terminate at either subsonic or supersonic points: for $\beta_{CJ} < \beta < \beta_{SS}$, $M > 1$; for $\beta_{SS} < \beta < \beta_{detach}$, $M < 1$. Generally $\beta_{SS} \approx \beta_{detach}$; consequently the range of weak overdriven solutions with $M < 1$ is small. Weak underdriven solutions terminate at supersonic points, $M > 1$.

The conditions under which these solution classes, each of which satisfies the conservation principles and entropy inequality, could exist in nature is a question which has not been completely answered. A first step is to consider the resolved steady reaction zone structures and examine solution trajectories from an initial state to an equilibrium state in phase space. For a given kinetic scheme, this will disqualify certain classes of solutions. Those that remain should be subjected to the more rigorous test of hydrodynamic stability. What should result is a knowledge of the initial and boundary conditions which are necessary for a solution to exist. Based on analogies with inert theory which show that the existence of a strong or weak oblique shock depends on the downstream boundary conditions, it is hypothesized that there may be boundary conditions for each class of oblique detonation to exist. Given that in the course of its travels, both an ODWE and ram accelerator may encounter boundary conditions suitable for each class of oblique detonation, it stands to reason that each class should be subjected to systematic study.

With this philosophy in mind Powers and Stewart studied steady reaction zone structures. With the one-step kinetic model and for an oblique detonation characterized by a straight lead shock, it was shown that the reactive Euler equations admit strong, weak overdriven, and CJ solutions but do not admit weak underdriven solutions. The extension of Powers and Gonthier, 1992a, allowed for a two-step reaction with the first step exothermic and the second endothermic. For convenience, they define an equivalent Q for two-step kinetics,

$$Q = \lambda_1 q_1 + \lambda_2 q_2, \tag{10}$$

where $\lambda_1, \lambda_2, q_1$, and q_2 are the reaction progress $(0 \leq \lambda_1, \lambda_2 \leq 1)$ and heat release associated with the first and second reactions, respectively. It was shown that with such a model, steady solutions for all three classes are available and furthermore that the weak underdriven solution can be obtained for eigenvalues of shock angle.

Shock polars and reaction trajectories for all three classes are shown in Figure 10. Here $q_1 = 100$, $q_2 = -75$ so that at complete reaction $Q = 25$, as in Figure 9. However, due to the variable reaction rates, Q can and does take on larger values within the reaction zone. The results give the two-dimensional extension to the one-dimensional case described in detail by Fickett and Davis, pp. 168-173, which admits eigenvalue solutions. As such, straightforward analogies exist. It can be shown that lines of constant β correspond to Rayleigh lines and the shock polars correspond to partial reaction

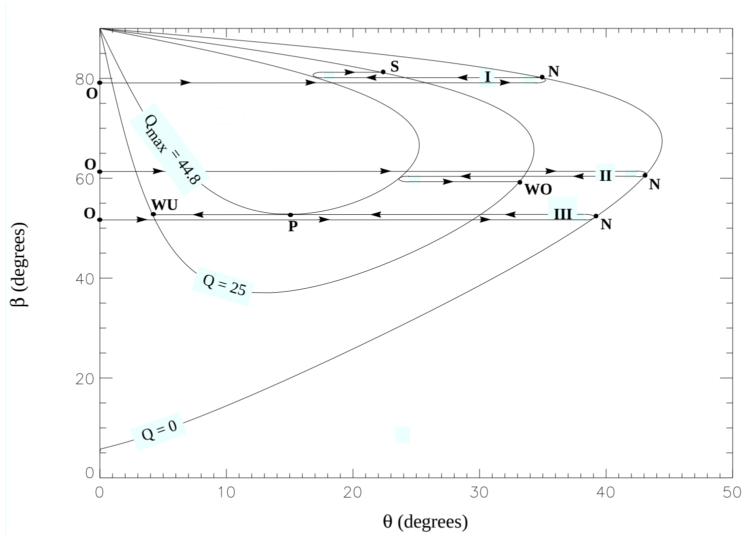

Figure 10: Inert ($Q = 0$), intermediate ($Q = 44.8$), and complete reaction ($Q = 25$) polars with reaction trajectories for strong (**I**), weak overdriven (**II**), and weak underdriven (**III**) cases, from Powers and Gonthier, 1992a.

Hugoniot curves. For each class, strong (labelled **I**), weak overdriven (labelled **II**), and weak underdriven (labelled **III**), the reaction proceeds by shocking the fluid from the inert state **O** to the shocked state **N**. The reaction then proceeds along a line of constant β (on either **I**, **II**, or **III**), through the curve of maximum heat release (in this case $Q_{max} = 44.8$) until the reaction is complete at either the strong point **S**, the weak overdriven point **WO**, or the weak underdriven point **WU**. The state **WU** is accessible upon passage through the saddle point **P**. For this scenario it was shown that the eigenvalue wave angle is the minimum wave angle for a steady solution; thus, the CJ wave angle, which is lower, places an overly restrictive lower bound on oblique detonation wave angle. It was also inferred that more detailed kinetics could yield correspondingly more complex conditions for the existence of steady waves.

Powers and Stewart also considered rotational solutions in the asymptotic limit of high incoming Mach number, M_0. Here the depiction of Figure 5 was mathematically confirmed. The solution procedure was to linearize the equations in the limit of high Mach number, write them in characteristic form, and construct a solution which simultaneously satisfied the RH jump conditions and a kinematic downstream wall boundary condition. The rather detailed solution can be expressed as an infinite series.

Grismer and Powers then compared the rotational asymptotic solutions to full numerical solutions. The numerical solution was obtained with the RPLUS code (Shuen and Yoon, 1989), in development at the NASA Lewis Research Center, using standard available features to simulate the flow. A series of comparisons was performed in which the only variables were the incoming Mach number and the heat release. For zero heat release, in which case the exact solution is available, it was deduced that at low supersonic Mach number the difference in the predictions of the asymptotic and numerical method was primarily attributable to the error in the asymptotic method, while at high Mach number the difference was primarily due to the numerical method. Similar results were inferred for flows with heat release in which there is no exact solution with which to compare. This is expected as the asymptotic solution should become more accurate as the ratio of heat release to flow kinetic energy becomes smaller while in the same limit, a point is reached when numerical errors overwhelm the effects of heat release. For very high Mach numbers, the numerical results become notably distorted while for very low (but still supersonic) Mach numbers, the same can be said for the asymptotic results.

A comparison of asymptotically and numerically predicted dimensionless pressure contours is shown in Figure 11. Here $M_0 = 20$ and $q = 10$. Assuming the ambient fluid is at temperature $300K$, this corresponds to a dimensional heat release of $\tilde{q} = 0.861 MJ/kg$. The numerical values assigned to the three contours correspond to $\tilde{P} = 85.484 bar$, $85.852 bar$, and $86.168 bar$ if the ambient pressure is assumed to be $\tilde{P}_0 = 1.000 bar$. In this case it is seen that there is qualitative and quantitative agreement in the two methods' predictions. In order to achieve this agreement, it was necessary to study Mach numbers in a regime far from where the ideal gas, constant property model is valid. Such a step can be justified given that the purpose of this study was to develop a benchmarking procedure for reacting flow codes.

In an effort to better relate these models to propulsion applications, Powers and Gonthier, 1992b, give a methodology to study of the configuration of Figure 7 along with a simple, non-rigorous anal-

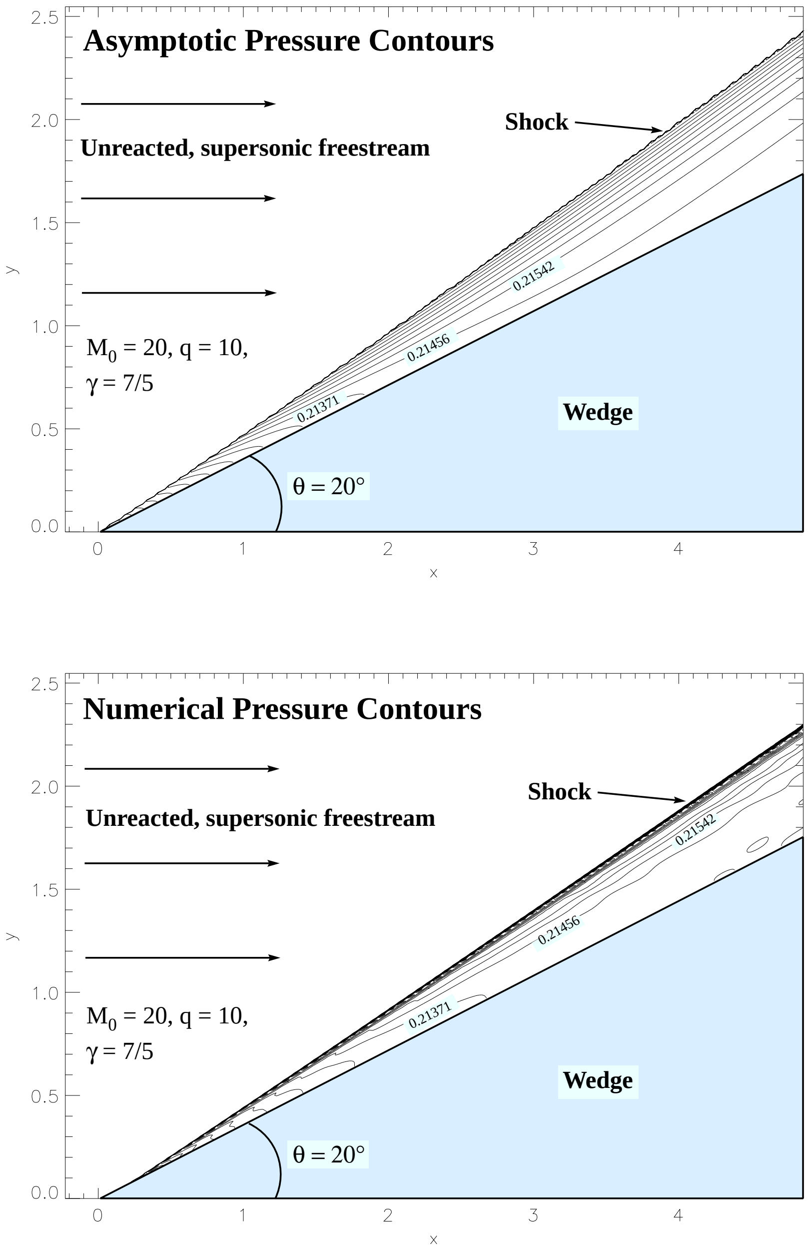

Figure 11: Dimensionless pressure contours predicted by asymptotic and numerical analysis, from Grismer and Powers, 1992.

ysis. The analysis divides the flow into six zones: 1) a pre-shocked region, 2) a post-shocked region, 3) a Prandtl-Meyer rarefaction region, 4) a post-rarefaction region, 5) a post-flame sheet region, and 6) a post-shock region. The transition from one zone to another is described by algebraic jump relations. The flame sheet is assumed *ad hoc* to be oriented normal to the lee wedge surface at such a location that a force balance exists. A thermal explosion theory is used to fix the flame location as a function of incoming Mach number.

Plausible results are obtained which are summarized in the bifurcation diagram of Figure 12. Here predictions of flight Mach number are plotted as a function of equivalence ratio $q/\tilde{Q}$ where $\tilde{Q}$ is the heat release associated with stoichiometric hydrogen-air combustion at atmospheric conditions. Below a critical heat release value, the heat release is insufficient to overcome wave drag, and there is no steady solution. Above this critical value, two solutions exist. The lower branch is unstable in a quasi-static sense in that a small perturbation of velocity gives rise to a force which moves the projectile away from equilibrium while on the upper branch a small perturbation in velocity gives rise to a restoring force. Thus one reaches the intuitively satisfying conclusion that an increase in energy released in combustion gives rise to an increase in flight speed. In making such stability conclusions, neither the inertia of the projectile or fluid has been taken into consideration. Finally, no correlation between the steady flight speed and CJ Mach number was found.

It is emphasized that these conclusions are based upon *ad hoc* modeling assumptions and that a more detailed study is required before ascribing any particular value of the predictions. Currently the author is studying numerical solutions to the flow over the double wedge which remove these difficulties.

4. Recommendations

In conclusion, it is suggested that simple models continue to be used effectively to address questions of relevance to the propulsion community. Though they cannot serve as a substitute for either comprehensive models or experiments (both of which have their difficulties), they can be useful guides for understanding. Examples of new configurations which could be considered are the reactive flow over a double-wedge at an angle of attack, flow considering the effect of cowling, conical geometries, reactive flow through a Busemann bi-

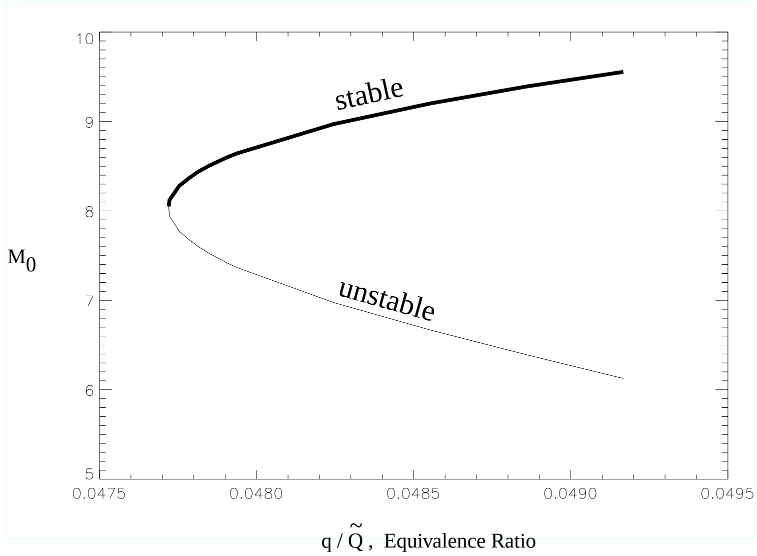

Figure 12: Bifurcation diagram for steady-state flight Mach number versus equivalence ratio, from Powers and Gonthier, 1992b.

plane, and flow over a thin airfoil. Simple model extensions which deserve study include modeling of realistic chemistry with rationally reduced kinetic mechanisms, the inclusion of boundary layer effects, and the inclusion of inertial effects. In performing such studies, a fuller comprehension may be attained.

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